To Adopt an Online Recommender System? A Manufacturer's Strategic Choice in a Dual-Channel Setting

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Abstract

The past years have witnessed the prosperity of online recommender systems. Some firms have utilized such systems to promote their products and services whereas others have not. To reveal the motivation to use recommender systems, we consider a dual channel setting in which a manufacturer sells his products to consumers in a direct-sale channel and also uses a wholesale channel to sell via an online retailer. The manufacturer decides on whether to use an online recommender system in the direct-sale channel or not, for both the case of the retailer's adoption of the recommendation system and the case of no recommendation in the wholesale channel. Considering both the cost-per-sale (CPS) and cost-per-click (CPC) payment schemes in the context of the recommender system, we examine whether it is optimal for the manufacturer to adopt the recommendation service under two distinct scenarios: when the retailer utilizes the recommender system and when the retailer does not. Our game-theoretic analysis exposes that, if the retailer does not adopt the recommender system under both the CPS and CPC payment schemes, then the manufacturer's system adoption decision depends on the recommendation strength and cost. We also find that the manufacturer can benefit from the CPS payment with a sufficiently low recommendation cost. When the retailer adopts the recommender system, the manufacturer's optimal strategy is to abandon the recommender system under any payment scheme, and the manufacturer reduces his wholesale price when the recommendation strength increases.

Keywords: Channel management; Recommender system; Pricing; Payment scheme; Game theory.

1 Introduction

Over the past decade we have observed a boom in the e-commerce industry as well as a fierce competition among online firms. To succeed in the market, more and more companies resort to recommender systems to drive traffic to them for higher online sales. According to product and service features, recommender systems can capture consumer purchasing behaviours and use algorithms to identify

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consumer preferences and interests, and then recommend products to target consumers. This can help online firms attract consumers and thus increase sales. For instance, Amazon announced that 35% of sales comes from its recommender system (Kumar and Hosanagar 2019).

There are various types of recommender systems, such as those deployed by retailers themselves, search engines and third-party price comparison websites or software. The first can be referred to as self-built recommender systems, and the latter two can be referred to as independent third-party recommender systems. With the rapid growth of e-commerce, many retail platforms have deployed their own recommender systems to facilitate consumer purchases. For example, Amazon's recommendation system is mainly based on collaborative filtering algorithms, which analyse the user's shopping history, browsing behaviour and other data to recommend products that may be of interest. In our model, we pay special attention to independent third-party recommender systems, including search engines and price comparison recommendation shopping sites. For example, search engines such as Baidu and 360 help firms gain higher exposure and click-through rates on search engines through precise keyword positioning and bidding strategies (see, `https://esm.baidu.com/product/`). Shopping.com, as an independent comparison shopping website, provides users with product comparison and recommendation services to help them make purchasing decisions among many options. It currently has partnerships with retailers such as eBay and KaTo (see, `https://www.shopping.com/`).

For retailers deploying their own recommender systems, it recommends products to consumers already on its own platform. The adoption strategy of the recommender system in this case depends only on the incremental costs and benefits. For independent third-party recommender systems, on the other hand, it is not limited to the platform itself, but recommends to all consumers in the marketplace. And due to the different decision makers, the adoption strategy of the recommender system is affected by the interaction of the game among the supply chain members. In addition, unlike the platform's self-built recommender system, the third-party recommender system is chargeable. Our research model considers the characteristics of these independent third-party recommender systems and analyses how they influence the strategic choices of the dual-channel manufacturer. We argue that independent recommender systems are able to provide more objective and diverse product recommendations due to their independence, which is valuable for both consumers and suppliers. However, the adoption strategy of a recommender system is not always advantageous for retailers or manufacturers because of expensive recommendation cost.

The recommender system charges any firm that uses the system to attract consumers under the cost-per-sale (CPS) or cost-per-click (CPC) payment schemes, which are reflected and analysed in our model. Under the CPS scheme, the recommender system calculates the firm's payment based on the number of incidents that consumers not only click product links but also decide to buy. The CPS payment scheme has been used by the Taobao Spreader and the Maixuan. We learn from the Sohu.com, a Chinese web portal and online media company, that on the basis of product selection, website construction, and marketing, combined with big data and social media, the Maixuan helps small- and medium-sized sellers easily carry out full-domain marketing strategies (see, `https://news.sohu.com/a/565332633_121418819`). Under the CPC scheme, the payment calculation depends on consumer clicks only (Chen et al. 2007). The CPC scheme has been adopted by the Baidu Marketing, Shopping.com, and Yahoo. More specifically, the Baidu marketing, developed based on the Baidu search engine in China, can realize the intelligent product delivery for users and improve product exposure rate. The Shopping.com provides users with easy-to-use search tools, engaging content, and time-saving navigation, and collects millions of user-generated product and merchant reviews which helps users make purchase decisions. Yahoo, an online platform based on the recommender system in the US, realizes recommendation via search engines, e-mails, websites or Apps, etc.

As any online firm has a discretion to adopt or not to adopt a recommender system which chooses between the CPS and the CPC payment schemes, it is naturally necessary, and interesting, to investigate an online firm's motivation to adopt the recommender system if the system implements the CPS or CPC payment scheme. Moreover, the online firms' decisions on whether to choose a recommender system or not also influence their supply chain partners.

We recognize that many scholars have confirmed through empirical studies that recommender systems have a significant impact on enhancing product exposure and consumer purchase intention (Adomavicius et al. 2018; Lee and Hosanagar 2021). These studies show that recommender systems play an important role in e-commerce, which highlights the need for research on recommender systems. Our study explores the mechanism of recommender systems' influence on market demand from the perspective of theoretical modelling, which complements previous empirical studies. Inspired by Li et al. (2018), who found that recommender systems may harm retailers that use recommender systems because retailers may focus only on themselves and ignore the strategies of upstream manufacturers, we consider the impact of recommender systems on the retailer along with the manufacturer's strategy and investigate the win-win conditions for the manufacturer and retailer. Moreover, Zhou et al. (2022), Yang and Gao (2017) and Zhou and Zou (2023) investigated the problem of adoption strategies for recommender systems in competitive environments through game theoretic models. In particular, Yang and Gao (2017) developed a game-theoretic model to study a retailer's recommendation strategy and found that the recommender system can coordinate the supply chain by mitigating channel conflicts. Unlike Yang and Gao's (2017) study, we not only focus on the retailer's recommender system adoption strategy problem, but also consider the upstream manufacturer's decision, in which the manufacturer and the retailer not only have a competitive-cooperative relationship in the channel, but also have the possibility of free-riding on the adoption of recommender systems. In addition, Yang and Gao (2017) and Zhou and Zou (2023) both study whether retailers build their own recommender systems, while we consider the adoption strategy problem of third-party recommender systems, which undoubtedly increases the complexity of the supply chain relationship. Existing literature about payment schemes of recommender systems most ignore the joint pricing decision of products and services (Yang et al. 2014; Liu and Mookerjee 2018, Chakraborty et al. 2021). Considering the payment schemes of third-party recommender systems, we aim to fill the research gap in this area by investigating the strategic game relationship among manufacturers, retailers, and third-party recommender systems through theoretical modelling.

In summary, we provide a theoretical framework for analysing the strategic interactions between the manufacturer and retailer in a dual-channel supply chain over product recommendation strategies using a game-theoretic model, in which the manufacturer can endogenously adopt recommender systems in order to increase market shares. Through the game-theoretic model, we not only derive the equilibrium pricing of the supply chain members, but also solve the win-win situations between the manufacturer and retailer in terms of their recommender adoption strategies. Since we consider the case of third-party recommendation, the recommendation payment scheme becomes a mandatory consideration. We therefore theoretically model the payment scheme of recommender systems, which has not been considered in previous empirical studies.

In this paper, we consider a dual channel system in which a manufacturer serves consumers in both a direct sale channel and a wholesale channel that is operated by a retailer. Under different recommendation strategies of the retailer, the manufacturer decides whether to adopt the recommendation service provided by a recommender system who may use the CPS or CPC payment scheme. We accordingly analyze seven scenarios as follows: (i) only the retailer adopts the recommender system with the CPS payment scheme (SN); (ii) both adopt the recommender system with the CPS payment scheme (SS); (iii) only the retailer adopts the recommender system with the CPC payment scheme (CN); (iv) both adopt the recommender system with the CPC payment scheme (CC); (v) no firm adopts the recommender system (NN); (vi) only the manufacturer adopts the recommender system with the CPS payment scheme (NS); (vii) only the manufacturer adopts the recommender system with the CPC payment scheme (NC). For each scenario, we analyze a game-theoretic model for the manufacturer, retailer and recommender system, with a focus on the manufacturer's strategic choice between the adoption and no adoption of the recommender system which uses either the CPS and CPC payment scheme. We also investigate the manufacturer's and the retailer's win-win situations upon the recommendation strategies under the CPS and CPC payment schemes.

Our findings underscore the significance of recommendation strength and cost in the manufacturer's recommender system adoption strategy. When the retailer adopts the recommender system, irrespective of the CPS or CPC payment scheme, the manufacturer's optimal strategy is to abandon the recommender system in the direct channel. What's more, the impact of a payment scheme on the manufacturer's recommendation strategy is that the manufacturer always prefers the CPS payment scheme if only one party, either the manufacturer or retailer, adopts the recommender system. However, if both firms adopt the recommender system, the manufacturer has a preference on the CPS payment scheme when the recommendation cost is sufficiently small, and prefers the CPC payment scheme otherwise. When the retailer refrains from adopting the recommender system, regardless of whether the recommender system uses the CPS or CPC payment scheme, a higher unit recommendation cost leads the manufacturer not to choose the recommender system. However, as the recommendation cost becomes moderate, the manufacturer's decision depends on the recommendation strength.

We also investigate the win-win situations for the manufacturer and the retailer, and find that the two firms can reach a win-win situation where neither of them adopts the recommender system when the recommendation strength is sufficiently low. For a moderately high recommendation strength and a relatively low recommendation cost, both firms prefer the scenario where only the manufacturer adopts the recommender system. It is interesting to learn that the win-win situation cannot be achieved when only the retailer adopts the recommender system, which indicates that the retailer is reluctant to permit the manufacturer to take a "free ride". Regarding the impact of CPS and CPC payment schemes in equilibrium, we observe that the CPC payment scheme requires a lower recommendation price so that firms can afford lower transfer costs. Our findings expose the insights regarding the adoption of recommender system for the manufacturer in a dual channel supply chain, providing novel perspectives on pricing strategies, preferences, and recommender system operations.

The rest of this paper is organized as follows. Section 2 presents the literature review. In Section 3 we obtain the equilibrium results for scenarios SN, SS, CN, and CC when the retailer adopts the recommender system under the CPS and CPC payment schemes. In Section 4, we analyze scenarios NN, NS, and NC in which the retailer does not adopt the recommender system. Then, we investigate the win-win situations for the manufacturer and the retailer in Section 5. Section 6 is dedicated to the analysis of extended modeling. This paper ends with a summary and concluding remarks in Section 7. We provide additional definitions and results in Appendix A, analyze the preferences of retailers and recommender systems in Appendix B, and relegate all proofs to Appendix C.

2 Literature Review

Our paper is related to three streams of extant publications, which are recommender systems, recommendation pricing models, and channel competition.

2.1 Recommender Systems

Recommender systems often play an important role in e-commerce with the development of information technology. Self-deployed recommender systems by retailers have been widely studied by scholors. Adomavicius et al. (2018) confirmed that recommender systems affect consumers' willingness to pay through empirical studies. Through a large-scale field experiment study, Lee and Hosanagar (2021) found that recommender systems can attract more views and improve conversion rate among consumers for all products, but this increase is moderated by product attributes and review ratings. However, they do not provide the impact of recommender systems on consumer or market demand from a modelling perspective.

Considering uniform pricing and differential pricing strategies, Zhou et al. (2022) studied the impact of recommender systems on the competition between store brands and national brands, and they also further analyzed consumers' searching behaviors under recommendation. They found that the introduction of recommender systems by store brands generally benefits them at the expense of national brands, particularly when the recommendation strength is moderate; however, high recommendation strength can lead to increased costs and reduced profits for store brands. Yang and Gao (2017) developed a game-theoretic model to investigate a retailer's recommendation strategies with two manufacturers. Results show that recommender systems can coordinate the supply chain by alleviating channel conflict even though a higher recommendation strength may hurt the manufacturer and the retailer. Both of them use game theoretic methods to deeply investigate the impact of self-deployed recommender systems on firms and consumers in competitive scenarios but ignored the scenario of third-party recommender systems. The classic feature of the payment scheme of such recommender systems is the innovation of our work.

However, recommender systems are not always beneficial to supply chain members. Li et al. (2018) built a model that contains two competing manufacturers and a common retailer. They found that recommender systems hurt the retailer who use recommendation, because the retailer may focus on itself but ignores the upstream manufacturers' strategies, and the situation becomes worse when the cost for developing recommender systems is high. Shi and Raghu (2020) investigated product recommendation strategies considering consumer taste and product quality interaction. It is interesting to note from that paper that when recommender systems cannot discern product types, recommendation strategies are not optimal. Zhou and Zou (2023) mainly explored how personalised product recommendation systems (e.g., Amazon) affect competition among third-party sellers on the platform, through a game-theoretic approach. The results show that sellers strategically adjust their prices to gain recommendations, and that this recommendation competition incentivises sellers to set intermediate prices to maximise the platform's profit from marginal consumers. Referring to Zhou and Zou (2023), we derive market demand from the consumer utility function and consider the effect of consumer heterogeneity in our extension. In contrast, we explore the problem of product recommendation in a dual-channel supply chain, where the manufacturer can endogenously adopt recommender systems to increase market share. In addition, our paper considers the case of third-party recommendations, so the recommendation payment scheme becomes a mandatory consideration.

The publications reviewed above are concerned with how recommender systems affect consumer purchase decisions and supply chain members' strategies. Nonetheless, little research has considered the competition between channels. This differs from the practice in which a manufacturer usually sells its products in different channels such as direct channels and wholesale channels. In addition, they all examine the self-deployed recommender systems but ignore the existence of third-party recommender systems. In this paper, we consider a manufacturer selling products through his own direct-sale channel and a wholesale channel to investigate the impact of a recommender system on the manufacturer's optimal strategy in the market which is divided into a traditional market segment and a recommended market segment. Moreover, we take into account recommender system's different payment schemes, i.e., the CPC vs. CPS payment schemes.

2.2 Recommendation Pricing Models

The previous literature emphasises the importance of self-deployed recommender systems in driving sales. However, none of them studied third-party recommender systems whose classical features are payment schemes, which is the innovation of our work. This subsection focuses on research in the area of pricing models, including not only the pricing of products, but also the pricing of third-party recommendation services. This literature below further complements the research on recommender systems.

Pricing is an important decision in any supply chain in which researchers usually consider product pricing and service pricing. For the product pricing decision, many researchers have studied the pricing problems under the channel competition. Lu et al. (2014) proposed that a firm can use a pricing strategy based on quantity discounts to increase sales. Kireyev et al. (2017) built a game-theoretic model to learn the impact of self-matching strategy on its pricing decision under different cases, including a monopolist, two competing multichannel retailers, and a mixed duopoly. Fisher et al. (2018) conducted an empirical model to capture consumer choices from multiple retailers and proposed a best-response pricing strategy, which increases revenue by 11%. In particular, Jiang et al. (2015) examined e-retailers' price discounts and recommendation strategies and found that sufficiently high price discounts can attract customers, but they are also prone to losses. In contrast, the gains from product recommendations can offset the losses caused by discounts. Regrettably, they did not consider the pricing strategy regarding recommendations. For the service pricing, researchers have examined many payment types such as cost-per-action (CPA), CPC, cost-per-impression (CPM), CPS, et al (Chen et al. 2017, Liu and Mookerjee 2018, Chakraborty et al. 2021).

With respect to payment schemes and pricing for recommendation services, Yang et al. (2014) applied the Bayesian estimation to conduct a empirical study and explore the effects of competition on the click volume and CPC. Najafi-Asadolahi and Fridgeirsdottir (2014) considered uncertain conditions (i.e., uncertain demand, uncertain traffic on websites and visitors' uncertain click behaviors) when they explored a publisher's optimal choice. If a principal constrains click-through-rate, Mookerjee et al. (2017) commented that it is inadvisable to present ads to every visitor, and service providers should show ads to those visitors who are likely to click only. However, the CPC payment scheme is not always optimal because it may induce illegitimate users to make click fraud behaviors. Chen et al. (2015) studied click fraud identification processes under the CPC payment scheme and found that the identification cost is usually so high that any improvement of the precision of investigation technology hurts the service provider. Hu et al. (2016) used incentive contracts to explore the impact of the CPC and CPA payment schemes on entities' risk sharing and efforts. The result is that the CPA payment scheme dominates the CPC payment scheme from the perspective of incentivizing the publisher's effort, even though the CPA scheme may cause an adverse selection problem.

The above review exposes that the extant publication have studied different pricing strategies from many perspectives. However, they considered neither the joint product and service pricing decision nor the impact of channel competition on firm's pricing strategy. Our contribution is summarized in three aspects. First, we consider a vertical supply chain where a manufacturer sells products not only via its direct-sale channel but also via a wholesale channel. Second, we analyze the third-party service provider's payment strategy and service pricing problems. We also examine the manufacturer and the retailer's pricing strategies and their preferences on the recommender system decision.

2.3 Channel Competition

With the development of e-commerce, many manufacturers expand their sales channels to gain more market shares. As a result, the competition between different channels has attracted great attention from both academics and practitioners (Chiang et al. 2003, Li 2018, Hotkar and Gilbert 2021). In reality, in addition to existing retail channels, suppliers usually open direct channels for sales, i.e., the encroachment of suppliers results in a dual-channel supply chain (Maruthasalam and Balasubramanian 2023). Iyer (1998) investigated a manufacturer's channel distribution strategy when a retailer competes with the manufacturer in price and other services. Yoo and Lee (2011) studied the impact of a manufacturer's direct-channel entry, and found that the entry does not always enhance consumer welfare but can benefit the retailer sometimes. A manufacturer's direct channel entry mitigates double marginalization but the results are different when there is information asymmetry. For example, Li et al. (2014) found that a supplier's direct channel entry leads to a reseller's deviated signals. That is, the retailer can deliberately reduce his order quantity, which amplifies double marginalization. Gao and Su (2017) examined a retailer's channel operation problem in which the retailer provides options for consumers to buy online and pick up in store (BOPS). The result shows that BOPS can alleviate incentive conflicts between channels but is not suitable for the products that are sold well in stores. Xu et al. (2022) investigated the impact of consumer channel preference and free-riding behavior on optimal sales effort levels and pricing decisions in a dual-channel supply chain. Our study differs from theirs in that we study the issue of optimal pricing and recommendation strategies in a dual-channel supply chain and analyze the behavior of a manufacturer that does not use recommendations itself but obtains them indirectly through free-riding from a downstream retailer. In addition, a manufacturer sometimes sells its products through a platform. Ha et al. (2022) considered three sale formats: agency selling, reselling, and dual channel sales, and analyzed the impact of channel competition on a supplier and a platform. Abhishek et al. (2016) explored when an e-tailer prefers to use the agency sale or reselling strategy, and found that the preference depends on the competition between e-tailers.

The above review indicates that the problem for channel competition is worth studying. The literature examined when and how to operate multiple channels. In practice, many manufacturers and retailers have already launched omni channels, which make them compete with each other with their services. In a dual-channel environment, the relationship between the manufacturer and the retailer is cooperative and competitive. Recommender systems can increase the sales potential of each channel by attracting consumers through recommendations, thereby expanding market coverage. Therefore, in the competitive dual-channel environment, manufacturers or retailers have an incentive to sell their products through the adoption of recommender systems and thus earn greater profits. In this paper, different from relevant publications, we focus on the recommendation strategies of a manufacturer who sells products through his own direct channel and a wholesale channel with an online retailer and analyze the influence of recommendation strength and cost on the manufacturer's recommendation strategies and the two firms' optimal pricing decisions. In addition, considering the co-opetition relationship between the manufacturer and retailer, we examine the impact of payment schemes, i.e. CPS and CPC, on the manufacturer's decision making and preferences.

3 The Models when the Retailer Adopts the Recommender System

We consider an online operating system consisting of a manufacturer, a retailer, and a recommender system who may use CPS or CPC payment scheme. The manufacturer sells its products to consumers in a market through both an online direct-sale channel and an online wholesale channel. In the direct-sale channel (e.g., the manufacturer's official website), the manufacturer sells the products directly to consumers at a unit retail price p_m, whereas in the wholesale channel, the manufacturer sells products at a unit wholesale price w to the online retailer who then sells them to consumers at a unit retail price

p_r. Such dual-channel setting is common in literature (e.g., Liu et al. (2021), Zhang et al. (2024)). The notations in this paper are summarized in Table 1.

Table 1: Notations and descriptions.

Notation	Description
U	Consumer utility in the traditional market
$\hat{U}$	Consumer utility in the recommended market when only one party uses recommender system
$\tilde{U}$	Consumer utility in the recommended market when both sides use recommeder system
α	Potential market size in the traditional market
$\hat{\alpha}$	Recommendation strength, i.e., potential market size in the recommended market
p_m	Direct-sale price set by the manufacturer
p_r	Retail price set by the retailer
p_e	Per unit recommendation fee set by the recommender system
w	Wholesale price set by the manufacturer
$D_i, i \in \{m, r\}$	Demand of direct-sale or wholesale channel in the traditional market
$\hat{D}_i, i \in \{m, r\}$	Demand of channel in the recommended market when only one party uses recommender system
$\tilde{D}_i, i \in \{m, r\}$	Demand of channel in the recommended market when both sides use recommeder system
θ	Substitutability between the direct-sale channel and the wholesale channel
c	Recommender system's unit recommendation cost
k	Coefficient of conversion rate
$\pi_i, i \in \{m, r, e\}$	Profits for the manufacturer, retailer and recommender system

In practice, online retailers can implement recommender systems to offer consumers personalized product recommendations (Gupta and Kumar 2020, Zhou and Zou 2023). Consumer behaviors can be categorized into two distinct types according to Wu et al. (2015), Yang and Gao , Zhou and Zou (2023). Some consumers are independently aware of a product's existence and attributes without relying on recommender systems, and their purchasing decisions remain unaffected by recommendations. These individuals can be categorized as informed consumers. In contrast, other consumers lack prior knowledge about the product and fully depend on recommender systems to discover and evaluate available options. Their purchasing behavior is often initiated by exploring recommended products and following purchase links provided by the system, thus classifying them as uninformed consumers. To examine the impact of recommender systems on consumers, akin to Ghose et al. (2007) and Wu et al. (2015), we categorize the market into two segments: traditional and recommended, based on the types of consumers. In the traditional market segment, consumers make purchasing decisions without relying on recommender systems, opting instead for either direct-sale or wholesale channels. In the recommended market segment, consumers utilize recommender systems to search for products and subsequently complete their purchases by clicking on provided purchase links. Intuitively, the greater the recommendation strength, the stronger the consumer preference for products in the recommended market. Similarly, the more consumers purchase through the recommender system, the larger the potential market size of the recommended market becomes. Here, we use α and $\hat{\alpha}$ to denote the preferences of consumers buying from the traditional market and the recommended market, respectively. Both α and $\hat{\alpha}$ incorporate the intrinsic value of products as an attribute. They are independent of each other, meaning they do not exert any influence on one another, and there is no restriction on their relative magnitudes.

Due to the variety of payment schemes for recommender systems, we consider two cases, i.e., CPS and CPC payment schemes. In this section, we investigate the following four recommendation scenarios where the retailer uses a recommender system: (i) the retailer adopts the recommender system in the wholesale channel but the manufacturer does not under the CPS and CPC payment schemes, which are referred to as scenarios "SN" and "CN", respectively; (ii) both the retailer and the manufacturer adopt the recommender system under the CPS and CPC payment schemes, which are referred to as scenarios "SS" and "CC", respectively. And in Section 4, we will explore three recommendation scenarios where the retailer does not use a recommender system. In each scenario, the manufacturer first decides on whether to adopt the recommender system or not. Secondly, the recommender system determines its unit service price p_e. Thirdly, the manufacturer determines both p_m and w. Finally, the retailer makes a decision on p_r (Wang et al. (2022)). Note that we also consider the case that the manufacturer determines w before the manufacturer and retailer simultaneous determines p_m and p_r in section 6.1, and the results show that our findings are robust. To distinguish our solutions in different scenarios, we use the superscripts "SN", "SS", "CN", and "CC" to represent equilibrium solutions for scenarios SN, SS, CN, and CC, respectively. Moreover, if the recommender system can offer both CPS and CPC payment schemes at the same time, we study the manufacturer and retailer's preferences in terms of payment schemes in section 6.4. In addition, we further investigate the preferences of the retailer and recommender system concerning CPS and CPC payment schemes in Appendix B.

3.1 Scenario SN: Only the Retailer Adopts the Recommender System under the CPS Payment Scheme

In this scenario, only the retailer adopts the recommender system who chooses the CPS payment scheme in the wholesale channel but the manufacturer does not adopt the recommender system in the direct-sale channel.

In the traditional market, each consumer can buy the manufacturer's product from either the direct-sale channel or the wholesale channel, which implies that these two channels are substitutable. We initially assume that all consumers purchase products exclusively from either the manufacturer or the retailer. In Section 6.2, we relax this assumption by introducing an outside option for consumers. We accordingly use $\theta \in [0, 1)$ to represent substitutability between these two channels. The channels are independent when $\theta = 0$ but they are completely substitutive when $\theta = 1$. Similar to Spence (1976), Cai et al. (2012), Wu et al. (2015), and Zhou and Zou (2023), for the traditional market, we develop the consumer utility function in quadratic form as

$$U = \sum\nolimits_{i=m,r} \left(\alpha D_i - p_i D_i - \frac{1}{2} D_i^2 \right) - \theta D_m D_r, \tag{1}$$

where α denotes consumers' preference for purchasing from the traditional market, D_m and D_r represent the demands in the direct-sale and wholesale channels, respectively. Furthermore, αD_i denotes the positive utility to consumers from the size of the potential market. The representative consumer pays $\sum_{i=m,r} p_i D_i$ if he has a sufficient budget. The term $D_i^2/2$ then represents the classical economic feature of diminishing marginal utility. In addition, due to θ representing the substitutability between these two channels, then $\theta D_m D_r$ can be seen as the competition between the two products in the market. The greater the intensity of competition is, the greater the substitutability of the product. As products become more substitutable, consumer utility tends to decrease. Moreover, in Section 6.3, we account for the heterogeneity of consumers' preferences across different channels. Although the utility function differs from that in the main model, the results remain robust.

Maximizing U in (1) yields the aggregate demands in direct-sale and wholesale channels as

$$D_m = \frac{(1-\theta)\alpha - p_m + \theta p_r}{1-\theta^2} \text{ and } D_r = \frac{(1-\theta)\alpha - p_r + \theta p_m}{1-\theta^2}.$$

In the recommended market, as mentioned before Section 3.1, we use $\hat{\alpha}$ to denote the recommendation strength and develop the consumer utility function in quadratic form similar to (1) as

$$\hat{U} = \hat{\alpha}\hat{D}_r - p_r\hat{D}_r - \frac{1}{2}\hat{D}_r^2, \tag{2}$$

and maximize it to find the demand in the recommended market as $\hat{D}_r = \hat{\alpha} - p_r$.

The recommender system (e.g., ShareASale, `https://www.shareasale.com/info/`) offers recommendation services to consumers in the wholesale channel, for which the retailer pays to the system a service fee $p_e f(\hat{D}_r)$, where p_e denotes the per unit recommendation fee and $f(\hat{D}_i), i = \{m, r\}$ denotes a recommendation payment scheme. Specifically, $f(\hat{D}_i) = \hat{D}_i$ under CPS payment scheme, and $f(\hat{D}_i) = \hat{D}_i/(k\hat{\alpha})$ under CPC payment scheme. Here, the recommender system gains $p_e\hat{D}_r$ from the sales of $\hat{D}_r$ products in the wholesale channel. Setting zero marginal production cost without loss of generality, we develop the manufacturer's, retailer's, and recommender system's profit functions as $\pi_m = w(D_r + \hat{D}_r) + p_m D_m$, $\pi_r = (p_r - w)(D_r + \hat{D}_r) - p_e f(\hat{D}_r)$, and $\pi_e = (p_e - c)f(\hat{D}_r)$, where c is the recommender system's unit recommendation cost. We then compute the prices in Stackelberg equilibrium as

$$\begin{aligned} p_m^{SN} &= \frac{1}{4}(\alpha(2-\theta) + \theta\hat{\alpha}),\ w^{SN} = \frac{-c(1-\theta^2) + \alpha(5-\theta-2\theta^2) - (3-\theta^2)\hat{\alpha}}{4(2-\theta^2)}, \\ p_r^{SN} &= \frac{c(1-\theta^2) - \alpha(-3+\theta+\theta^2) + (11-6\theta^2)\hat{\alpha}}{8(2-\theta^2)},\ p_e^{SN} = \frac{c(1-\theta^2) + \alpha(-3+\theta+\theta^2) + (5-2\theta^2)\hat{\alpha}}{2(1-\theta^2)}. \end{aligned}$$

Using the equilibrium prices, we can find the three firms' profits, as given as in Appendix A.1. For non-negativity of our outcomes, we require recommendation strength $\hat{\alpha} \in (\hat{\alpha}_l^{SN}, \hat{\alpha}_h^{SN})$ where $\hat{\alpha}_l^{SN}$ and $\hat{\alpha}_h^{SN}$ are defined as in Appendix A.1.

Proposition 1 *For scenario SN, we find that p_e^{SN}, p_m^{SN}, and p_r^{SN} increases with $\hat{\alpha}$, whereas w^{SN} decreases with $\hat{\alpha}$. In addition, both p_e^{SN} and p_r^{SN} increase with c, w^{SN} decreases with c, and p_m^{SN} is independent of c.*

We learn from Proposition 1 that the recommendation strength $\hat{\alpha}$ and cost c raise retail price and recommendation service price. Specifically, as recommendation strength and cost increase, the recommender system increases its service price to offset the costs. That is, the cost increment is transferred from the recommender system to the retailer who then raise the retail price. Moreover, the manufacturer opts for a higher direct-channel price in response to elevated recommendation strength, aiming not only to harmonize market prices but also to maximize profits. However, when the recommendation strength and cost are higher, the manufacturer adopts a strategy of reducing the wholesale price primarily to retain the retailer.

For scenario SN, we obtain the following results regarding the impacts of $\hat{\alpha}$ and c on the profits in Table 2.

Table 2: The impacts of recommendation strength and cost on the profits in scenario SN. Here, $\hat{\alpha}_i^{SN}$ $(i = 1, 2, 3, 4)$ are obtained in the Online Appendix C, $\hat{\alpha}_l^{SN}$, and $\hat{\alpha}_h^{SN}$ are defined as in Appendix A.1.

	When $\hat{\alpha}$ increases	When c increases
π_m^{SN}	$\uparrow$ if $\hat{\alpha} \in (\hat{\alpha}_1^{SN}, 1) \cap (\hat{\alpha}_l^{SN}, \hat{\alpha}_h^{SN})$; $\downarrow$ otherwise.	$\uparrow$ if $\hat{\alpha} \in (\hat{\alpha}_3^{SN}, 1) \cap (\hat{\alpha}_l^{SN}, \hat{\alpha}_h^{SN})$; $\downarrow$ otherwise.
π_r^{SN}	$\uparrow$ if $\hat{\alpha} \in (0, \hat{\alpha}_2^{SN}) \cap (\hat{\alpha}_l^{SN}, \hat{\alpha}_h^{SN})$; $\downarrow$ otherwise.	$\uparrow$ if $\hat{\alpha} \in (0, \hat{\alpha}_4^{SN}) \cap (\hat{\alpha}_l^{SN}, \hat{\alpha}_h^{SN})$; $\downarrow$ otherwise.
π_e^{SN}	$\uparrow$	$\downarrow$

Table 2 indicates that when the recommendation strength increases, the manufacturer's profit first decreases and then increases. Specifically, when the recommendation market is relatively small, the manufacturer's profit in the direct-sale channel decreases with the expansion of the recommendation market. However, with a moderate increase in recommendation strength, the manufacturer stands to benefit from recommendations, which can augment demand in the wholesale channel. Consequently, the recommender system thrives particularly when recommendation strength is heightened. This phenomenon arises because an escalation in recommendation strength motivates more consumers to make purchases via online recommendations, thereby amplifying profit margins. Furthermore, our findings indicate that the retailer can reap significant benefits from the recommendation service, especially when recommendation strength is not excessively high. However, in scenarios where recommendation strength is sufficiently large, the recommender system wields greater influence compared to cases where recommendation strength is relatively low. Consequently, the system increases its recommendation service price, which results in a profit decline for the retailer.

In addition, the firms' profits are dependent on unit recommendation cost c. If the recommendation strength is sufficiently large, the manufacturer is better off in his direct channel. Otherwise, the manufacturer subsidizes the retailer by providing a lower wholesale price, as shown in Proposition 1, with an increase in c. This leads the manufacturer to lose profit. The profitability of the recommender system exhibits a monotonic relationship with c. Regarding the retailer, although the recommendation cost results in a higher recommendation service price, the retailer determines a higher retail price to obtain a greater profit when the recommendation strength is sufficiently small. However, the escalation of the retail price may potentially dampen demand, thereby causing a decline in the retailer's profit as the recommendation strength reaches a certain threshold.

3.2 Scenario SS: The Manufacturer and the Retailer Adopt the Recommender System under the CPS Payment Scheme

For scenario SS, similar to the traditional market in scenario SN, the consumer utility function in the recommended market is

$$\tilde{U} = \hat{\alpha}\tilde{D}_m - p_m\tilde{D}_m - \frac{1}{2}\tilde{D}_m^2 + \hat{\alpha}\tilde{D}_r - p_r\tilde{D}_r - \frac{1}{2}\tilde{D}_r^2 - \theta\tilde{D}_m\tilde{D}_r, \tag{3}$$

where $\tilde{D}_m$ and $\tilde{D}_r$ represent the demands for the manufacturer and retailer, respectively, in the recommended market. Maximizing $\tilde{U}$ in (3), we have $\tilde{D}_m = [(1-\theta)\hat{\alpha} - p_m + \theta p_r]/(1-\theta^2)$ and $\tilde{D}_r = [(1-\theta)\hat{\alpha} - p_r + \theta p_m]/(1-\theta^2)$, and compute the three firms' profits as $\pi_m = w(D_r + \tilde{D}_r) + p_m(D_m + \tilde{D}_m) - p_e\tilde{D}_m$, $\pi_r = (p_r - w)(D_r + \tilde{D}_r) - p_e\tilde{D}_r$, and $\pi_e = (p_e - c)(\tilde{D}_m + \tilde{D}_r)$. We obtain the prices in Stackelberg equilibrium as

$$
\begin{aligned}
p_m^{SS} &= \frac{\alpha + 3\alpha\theta + c(3+\theta) + (17+3\theta)\hat{\alpha}}{8(3+\theta)};\ w^{SS} = \frac{\alpha(11+\theta) - c(3+\theta) + (-5+\theta)\hat{\alpha}}{8(3+\theta)};\\
p_r^{SS} &= \frac{\alpha(12-4\theta-\theta^2) + c(3+4\theta+\theta^2) + \hat{\alpha}(29+12\theta-\theta^2)}{16(3+\theta)};\ p_e^{SS} = \frac{\alpha(-5+\theta) + c(3+\theta) + (11+\theta)\hat{\alpha}}{2(3+\theta)};
\end{aligned}
$$

and also calculate the profits as given in Appendix A.2. To ensure non-negative outcomes, we require $c/2 < \alpha \leq 1$ and $\hat{\alpha} \in (\hat{\alpha}_l^{SS}, \hat{\alpha}_h^{SS})$ where $\hat{\alpha}_l^{SS}$ and $\hat{\alpha}_h^{SS}$ are defined as in Appendix A.2.

One can easily find that the impacts of recommendation strength and cost on prices are similar to those in Proposition 1.

In addition, to investigate the inpact of recommendation on the consumers' purchase decisions, we next conduct a sensitivity analysis of demand with respect to the recommendation strength, as shown in the following proposition.

Proposition 2 *$\hat{D}_m^{SS}$ and $\hat{D}_r^{SS}$ increase with $\hat{\alpha}$, whereas D_m^{SS} and D_r^{SS} decrease with $\hat{\alpha}$.*

Proposition 2 shows that the demand in the recommended market increases with the recommendation strength and the demand in the traditional market decreases with the recommendation strength.It is known that recommender systems work by recommending products to consumers, increasing product exposure and helping consumers discover products they may be interested in. As the recommendation strength increases, consumers are more likely to find the products they need, thus increasing the likelihood of purchase. However, as recommendation strength increases, more and more consumers will make purchases through the links of the recommender system, taking market share from the traditional marketplace. In other words, the greater the recommendation strength, the more consumers are affected by the recommender system. At this moment, some consumers will switch from the traditional market to the recommended market.

As for the impacts of $\hat{\alpha}$ and c on the firms' profits, we present our results in Table 3. The results in Table 3 indicate that the manufacturer's profit increases with the recommendation strength and cost, when the recommendation strength falls within a proper range. This implies that the recommender system can effectively assist the manufacturer in increasing profit when the recommendation strength and cost are sufficiently low. However, if either the recommendation strength or cost increases to a certain extent, the manufacturer may not be able to afford the expensive recommendation service cost, thus suffering from a profit loss.

Moreover, an escalation in recommendation strength within a sufficiently low range can yield advantages for the retailer. Nevertheless, as exposed in Table 2, when recommendation strength reaches a significant level, the recommender system gains enhanced bargaining power compared to scenarios with a lower recommendation strength. Consequently, the system is empowered to charge a higher

Table 3: The impacts of recommendation strength and cost on the profits in scenario SS. Here, $\hat{\alpha}_i^{SS}$ $(i = 1, 2, 3, 4)$, $\hat{\alpha}_l^{SS}$, and $\hat{\alpha}_h^{SS}$ are defined as in Appendix A.2 and Online Appendix C.

	As $\hat{\alpha}$ increases	As c increases
π_m^{SS}	$\uparrow$ if $\hat{\alpha} \in (0, \hat{\alpha}_1^{SS}) \cap (\hat{\alpha}_l^{SS}, \hat{\alpha}_h^{SS})$; $\downarrow$ otherwise.	$\uparrow$ if $\hat{\alpha} \in (0, \hat{\alpha}_3^{SS}) \cap (\hat{\alpha}_l^{SS}, \hat{\alpha}_h^{SS})$; $\downarrow$ otherwise.
π_r^{SS}	$\uparrow$ if $\hat{\alpha} \in (0, \hat{\alpha}_2^{SS}) \cap (\hat{\alpha}_l^{SS}, \hat{\alpha}_h^{SS})$; $\downarrow$ otherwise.	$\uparrow$ if $\hat{\alpha} \in (0, \hat{\alpha}_4^{SS}) \cap (\hat{\alpha}_l^{SS}, \hat{\alpha}_h^{SS})$; $\downarrow$ otherwise.
π_e^{SS}	$\uparrow$	$\downarrow$

service price, which causes a profit decline for the retailer. In fact, the retailer absorbs a portion of the escalated cost incurred by the recommender system. Consequently, a high recommendation strength can benefit the recommender system, facilitating a higher profitability by enabling recommendations of a broader array of products to consumers.

3.3 Scenario CN: Only the Retailer Adopts the Recommender System under the CPC Payment Scheme

Similar to scenario SN, the demand in the recommended market can be derived as $\hat{D}_r = \hat{\alpha} - p_r$, where $\hat{\alpha}$ represents the recommendation strength and p_r denotes the retail price. Under the CPC payment scheme, the recommender system's (e.g., Google's Adsense, `https://www.google.com/adsense/`) revenue depends on the number of consumers click on the retailer's website. We define the recommendation conversion rate as the ratio of the number of buyers (i.e., $\hat{D}_r$) to the number of consumers clicking product links through the recommender system. Letting N denote the number of consumers who click on the web (or simply, the number of consumer clicks), the recommendation conversion rate is expressed as $\hat{D}_r/N$. Naturally, this rate exhibits a positive correlation with $\hat{\alpha}$. Accordingly, we calculate the conversion rate as $\hat{D}_r/N = k\hat{\alpha}$, where $k > 0$ is the parameter representing the unit impact of $\hat{\alpha}$. Consequently, $N = \hat{D}_r/(k\hat{\alpha})$, and the retailer has to pay $p_e f(\hat{D}_r) = p_e\hat{D}_r/(k\hat{\alpha})$ to the recommender system. The profit functions is

$$\pi_m = w(D_r + \hat{D}_r) + p_m D_m,\ \pi_r = (p_r - w)(D_r + \hat{D}_r) - p_e f(\hat{D}_r), \text{ and } \pi_e = (p_e - c)f(\hat{D}_r).$$

We compute the prices in Stackelberg equilibrium as

$$\begin{aligned} p_m^{CN} &= \frac{1}{4}(\alpha(2-\theta) + \theta\hat{\alpha});\ w^{CN} = \frac{-c(1-\theta^2) + k\hat{\alpha}(\alpha(5-\theta-2\theta^2) + \hat{\alpha}(-3+\theta^2))}{4k\hat{\alpha}(2-\theta^2)}; \\ p_r^{CN} &= \frac{c(1-\theta^2) + k\hat{\alpha}(\alpha(3-\theta-\theta^2) + \hat{\alpha}(11-6\theta^2))}{8k\hat{\alpha}(2-\theta^2)};\ p_e^{CN} = \frac{1}{2}\left[c + \frac{k\hat{\alpha}(-\alpha(3-\theta-\theta^2) + \hat{\alpha}(5-2\theta^2))}{1-\theta^2}\right]; \end{aligned}$$

and calculate the three firms' profits as in Appendix A.3. For non-negativity of our results, we require $c \in (c_l^{CN}, c_h^{CN})$ where c_l^{CN} and c_h^{CN} are defined as in Appendix A.3.

Based on the equilibrium outcomes, we calculate the impacts of $\hat{\alpha}$ and c on the firms' prices; see Table 4 from which we find that an escalation in recommendation strength prompts the manufacturer to establish a higher direct-channel retail price, similar to that in Proposition 1. However, in contrast to Proposition 1, before the recommendation cost surpasses a certain threshold, the manufacturer reduces the wholesale price, potentially enticing the retailer to utilize the recommendation service. Moreover, the recommendation service price first decreases and then increases with the increase of recommendation strength. This phenomenon may be attributed to the necessity for the recommender system to entice retailers to adopt the recommendation service by initially offering a lower price when the recommended market is relatively small. Conversely, when the recommendation strength reaches a significant level, the system may capitalize on a higher recommendation price.

Table 4: The impacts of recommendation strength and cost on the profits in scenario CN. Here, c_i^{CN} $(i = 1, 2)$, $\hat{\alpha}_1^{CN}$, c_l^{CN}, and c_h^{CN} are defined as in Appendix A.3 and Online Appendix C.

	When $\hat{\alpha}$ increases	When c increases
p_m^{CN}	$\uparrow$	$-$
w^{CN}	$\uparrow$ if $c \in (c_1^{CN}, 1) \cap (c_l^{CN}, c_h^{CN})$; $\downarrow$ otherwise.	$\downarrow$
p_r^{CN}	$\uparrow$ if $c \in (0, c_2^{CN}) \cap (c_l^{CN}, c_h^{CN})$; $\downarrow$ otherwise.	$\uparrow$
p_e^{CN}	$\uparrow$ if $\hat{\alpha} \in (\hat{\alpha}_1^{CN}, 1)$ and $c \in (c_l^{CN}, c_h^{CN})$; $\downarrow$ otherwise.	$\uparrow$

The impact of recommendation cost on pricing decisions under the CPC payment scheme aligns with that under the CPS payment scheme. However, the effects of the recommendation strength on these decisions under the CPC payment scheme are non-monotonic. Table 5 shows the impacts of $\hat{\alpha}$ and c on the profits under scenario CN.

Table 5: The impacts of recommendation strength and cost on the profits in scenario CN. Here, c_i^{CN} $(i = 3, 4, \ldots, 10)$, $\hat{\alpha}_2^{CN}$, c_l^{CN}, and c_h^{CN} are defined as in Appendix A.3 and Online Appendix C.

	When $\hat{\alpha}$ increases	When c increases
π_m^{CN}	$\begin{cases} \uparrow, & \text{if } c \in (c_3^{CN}, c_4^{CN}) \cap (c_l^{CN}, c_h^{CN}); \\ \downarrow, & \text{otherwise.} \end{cases}$	$\begin{cases} \uparrow, & \text{if } c \in (c_8^{CN}, 1) \cap (c_l^{CN}, c_h^{CN}); \\ \downarrow, & \text{otherwise.} \end{cases}$
π_r^{CN}	$\begin{cases} \uparrow, & \text{if } c \in (c_5^{CN}, c_6^{CN}) \cap (c_l^{CN}, c_h^{CN}); \\ \downarrow, & \text{otherwise.} \end{cases}$	$\begin{cases} \uparrow, & \text{if } c \in (c_9^{CN}, 1) \cap (c_l^{CN}, c_h^{CN}); \\ \downarrow, & \text{otherwise.} \end{cases}$
π_e^{CN}	$\begin{cases} \uparrow, & \text{if } \hat{\alpha} \in (\hat{\alpha}_2^{CN}, 1) \text{ and } c \in \left(0, c_7^{CN}\right) \cap \left(c_l^{CN}, c_h^{CN}\right); \\ \downarrow, & \text{otherwise.} \end{cases}$	$\begin{cases} \uparrow, & \text{if } c \in (c_{10}^{CN}, 1) \cap (c_l^{CN}, c_h^{CN}); \\ \downarrow, & \text{otherwise.} \end{cases}$

Our analysis reveals that profits exhibit a non-monotonic relationship with the recommendation strength and cost under the CPC payment scheme. Specifically, only when the unit recommendation cost is moderate, the manufacturer and the retailer can benefit from a higher recommendation strength. This phenomenon arises due to the interplay between recommendation strength and cost. Specifically, when the unit recommendation cost is minimal, the wholesale price decreases whereas the recommendation service price increases with recommendation strength. This results in a detrimental impact on both the manufacturer and the retailer. Conversely, when the unit recommendation cost is sufficiently large, the recommendation service price becomes prohibitively large such that even a slight increase adversely affects both firms. In addition, when the recommendation strength is sufficiently small, the recommender system fails to garner acceptance from the retailer. However, as the recommendation strength increases, the system aids the retailer in bolstering sales. When the recommendation cost is low, the recommender system reaps profits from increasing recommendation strength. However, the recommender system is worse off if the recommendation strength and cost are sufficiently high, possibly because the inflated recommendation price dampens the retailer's incentive to adopt the system. Moreover, profits of the three firms are contingent on the unit recommendation cost. If the unit recommendation cost is sufficiently small, then the three firms' profits decrease with the unit recommendation cost. Otherwise, the three firms can benefit from a higher recommendation cost. The manufacturer and the retailer can still benefit from consumers' willingness to pay for a higher price.

3.4 Scenario CC: the Manufacturer and the Retailer Adopt the Recommender System under the CPC Payment Scheme

Similar to scenario SS, we have the profit functions as $\pi_m = w(D_r+\tilde{D}_r)+p_m(D_m+\tilde{D}_m)-p_e\tilde{D}_m/(k\hat{\alpha})$, $\pi_r = (p_r - w)(D_r + \tilde{D}_r) - p_e\tilde{D}_r/(k\hat{\alpha})$, and $\pi_e = (p_e - c)(\tilde{D}_m + \tilde{D}_r)/(k\hat{\alpha})$. The prices in Stackelberg equilibrium are

$$
\begin{aligned}
p_m^{CC} &= \frac{1}{8}\left(\frac{c}{k\hat{\alpha}} + \frac{3\alpha\theta+\alpha+3\theta\hat{\alpha}+17\hat{\alpha}}{\theta+3}\right);\ w^{CC} = \frac{1}{8}\left[\frac{\alpha(\theta+11)+(\theta-5)\hat{\alpha}}{\theta+3} - \frac{c}{k\hat{\alpha}}\right];\\
p_r^{CC} &= \frac{c(\theta^2+4\theta+3)-k\hat{\alpha}((\theta^2-12\theta-29)\hat{\alpha}+\alpha(\theta^2+4\theta-13))}{16(\theta+3)k\hat{\alpha}};\ p_e^{CC} = \frac{1}{2}\left[c + \frac{k\hat{\alpha}(\alpha(\theta-5)+(\theta+11)\hat{\alpha})}{\theta+3}\right].
\end{aligned}
$$

The three firms' profits are given as in Appendix A.4. We require $c \in (c_l^{CC}, c_h^{CC})$ (where c_l^{CC} and c_h^{CC} are defined as in Appendix A.4) for non-negativity of our results.

For scenario CC, we obtain the following results regarding the impacts of $\hat{\alpha}$ and c on the firms' prices. We find from Table 6 that the effects of the recommendation cost on the prices are identical to those in Proposition 1. However, the impacts of the recommendation strength on the prices differ from those as in Proposition 1. As the recommendation strength increases, the recommender system initially lowers the service price and then raises it. This indicates that the recommender system should strategically lower the service price to incentivize the recommendation adoption by both the manufacturer and the retailer, when the recommendation strength is relatively low. When the recommendation strength increases properly, the recommender system should set a higher service price to maximize its margin profit. Furthermore, when the unit recommendation cost is sufficiently low, not only the manufacturer raises the direct-channel retail price but also the retailer increases the retail price as the recommendation strength increases. This implies that the recommendation can alleviate price competition between the channels, when the unit recommendation cost is sufficiently low.

Table 6: The impacts of recommendation strength and cost on the profits in scenario CC. Here, c_i^{CC} $(i = 1, 2, 3)$, $\hat{\alpha}^{CC}$, c_l^{CC}, and c_h^{CC} are defined as in Appendix A.4 and Online Appendix C.

	When $\hat{\alpha}$ increases	When c increases
p_m^{CC}	$\uparrow$ if $c \in (0, c_1^{CC}) \cap (c_l^{CC}, c_h^{CC})$; $\downarrow$ otherwise.	$\uparrow$
w^{CC}	$\uparrow$ if $c \in (c_2^{CC}, 1) \cap (c_l^{CC}, c_h^{CC})$; $\downarrow$ otherwise.	$\downarrow$
p_r^{CC}	$\uparrow$ if $c \in (0, c_3^{CC}) \cap (c_l^{CC}, c_h^{CC})$; $\downarrow$ otherwise.	$\uparrow$
p_e^{CC}	$\uparrow$ if $\hat{\alpha} \in (\hat{\alpha}^{CC}, 1)$ and $c \in (c_l^{CC}, c_h^{CC})$; $\downarrow$ otherwise.	$\uparrow$

It is interesting to find that, different from the scenarios where only the manufacturer adopts the recommender system, the manufacturer reduces the wholesale price to compensate the retailer, when the recommendation cost is sufficiently low. Conversely, when the unit recommendation cost is sufficiently high, the manufacturer increases the wholesale price but lowers the direct-channel retail price with the increase of recommendation strength.

In addition, we obtain the impacts of $\hat{\alpha}$ and c on the firms' profits in Table 7, where we find that, when recommendation strength is sufficiently small and the unit recommendation cost is moderate, the manufacturer can benefit from a higher recommendation strength. However, when the recommendation strength becomes sufficiently large, the manufacturer can only benefit from a higher recommendation strength if channel competition remains limited. This occurs because in scenarios with a more intensive channel competition, the manufacturer may be compelled to lower prices to attract more consumers, which may lead to adverse outcomes for the manufacturer. Moreover, the retailer benefits from the increase of recommendation strength when the unit recommendation cost is moderate, similar to Table 5. A high recommendation strength is advantageous for the recommender system. In addition, similar to Table 5, if the unit recommendation cost is sufficiently high, both the manufacturer's and retailer's profits increase with the cost. Otherwise, if the cost is sufficiently low, the profits of both firms decrease with the cost. However, it is noteworthy that the profit of the recommender system consistently decreases with the cost, regardless of its magnitude.

Table 7: The impacts of recommendation strength and cost on the profits in scenario CC. Here, c_i^{CC} $(i = 4, 5, \ldots, 9)$, Ω_1, c_l^{CC}, and c_h^{CC} are defined as in Appendix A.4 and Online Appendix C.

	As $\hat{\alpha}$ increases	As c increases
π_m^{CC}	$\begin{cases} \uparrow, & \text{if } \hat{\alpha} \in \Omega_1 \text{ and } c \in (c_4^{CC}, c_5^{CC}) \cap (c_l^{CC}, c_h^{CC}); \\ \downarrow, & \text{otherwise.} \end{cases}$	$\begin{cases} \uparrow, & \text{if } c \in (c_8^{CC}, 1) \cap (c_l^{CC}, c_h^{CC}); \\ \downarrow, & \text{otherwise.} \end{cases}$
π_r^{CC}	$\begin{cases} \uparrow, & \text{if } c \in (c_6^{CC}, c_7^{CC}) \cap (c_l^{CC}, c_h^{CC}); \\ \downarrow, & \text{otherwise.} \end{cases}$	$\begin{cases} \uparrow, & \text{if } c \in (c_9^{CC}, 1) \cap (c_l^{CC}, c_h^{CC}); \\ \downarrow, & \text{otherwise.} \end{cases}$
π_e^{CC}	$\uparrow$	$\downarrow$

3.5 The Manufacturer's Strategic Choice between the Adoption and Abandonment of the Recommendation Service

To investigate whether the manufacturer should adopt the recommender system or not in the direct-sale channel, it is imperative to compare the manufacturer's profits under scenarios SN and SS when the recommender system uses the CPS payment scheme as well as those under scenarios CN and CC when the recommender system uses the CPC payment scheme. For non-negativity of prices and demands, we require $\hat{\alpha} \in F_a \equiv (\hat{\alpha}_l^{SN}, \hat{\alpha}_h^{SN}) \cap (\hat{\alpha}_l^{SS}, \hat{\alpha}_h^{SS})$, where $\hat{\alpha}_l^{SN}$, $\hat{\alpha}_h^{SN}$, $\hat{\alpha}_l^{SS}$ and $\hat{\alpha}_h^{SS}$ are defined as in Appendix A. We also require $c \in F_c \equiv (c_l^{CN}, c_h^{CN}) \cap (c_l^{CC}, c_h^{CC})$, where c_l^{CN}, c_h^{CN}, c_l^{CC} and c_h^{CC} are defined as in Appendix A.

Theorem 1 *We find that* $\pi_m^{SN} > \pi_m^{SS}$ *and* $\pi_m^{CN} > \pi_m^{CC}$.

According to Theorem 1, we find that, when the retailer adopts the recommender system under either CPS or CPC payment scheme, the manufacturer should not choose the recommender system's service. That is, the manufacturer always does not adopt the recommender system. This is mainly attributed to a high cost of the recommendation service. Opting for the recommender system in the direct-sale channel would necessitate an increase in the retail price, which may lead to the profit decline in the channel. Without using the recommender system, the manufacturer can instead concentrate on the wholesale channel where the retailer adopts the recommender system. Meanwhile, the manufacturer lowers his wholesale price to incentivize the retailer to adopt the recommender system. That is, the manufacturer prefers to take a "free ride" of product recommendation from the retailer rather than adopt the recommender system by himself.

3.6 The Manufacturer's Preference between the CPS and CPC Payment Schemes

As the recommender system may choose the CPS or CPC payment scheme, we explore whether the manufacturer benefits more from the CPS or CPC payment scheme.

Theorem 2 *We find that $\pi_m^{SN} > \pi_m^{CN}$, when the retailer adopts the recommender system but the manufacturer does not.*

Theorem 2 indicates that if the retailer adopts the recommender system but the manufacturer does not, then the manufacturer can benefit more when the recommender system uses the CPS payment scheme compared to when the system uses the CPC payment scheme. That is, the manufacturer benefits more from the CPS payment scheme, even if he does not adopt the recommender system, which can be viewed as the manufacturer's "free ride" activity. As demonstrated by Proposition 1 and Table 4, the escalation in recommendation costs prompts the manufacturer to mitigate possible losses incurred by the retailer by lowering the wholesale price. Moreover, the CPS payment scheme offers distinct advantages, notably a higher degree of certainty compared to the CPC counterpart, along with its effectiveness in countering click fraud behaviors. Consequently, by leveraging the retailer's recommender system, the manufacturer implicitly relies on the expectation that the retailer is shielded from unnecessary financial setbacks. This symbiotic relationship underscores the strategic importance of the CPS payment scheme in optimizing the manufacturer's benefits, even in scenarios where direct adoption of the recommender system may not be pursued.

Theorem 3 *When both the manufacturer and the retailer adopt the recommender system, we find that if $c < c_{m2}^{SC}$, then $\pi_m^{SS} > \pi_m^{CC}$; otherwise, $\pi_m^{SS} < \pi_m^{CC}$.*

Theorem 3 exposes that, when both firms use the recommender system, if the recommendation cost is sufficiently small, then the manufacturer prefers the CPS payment scheme to the CPC scheme; otherwise, the manufacturer prefers to choose the recommendation service under the CPC scheme. The result differs from that when only the retailer adopts the recommender system. We illustrate our results in Figure 1.

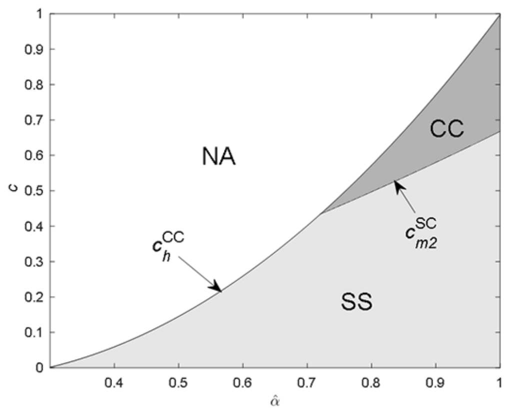


Figure 1: The manufacturer's preference between two payment schemes when both firms adopt the recommender system ($\alpha = 1, \theta = 0.8, k = 0.2$).

4 The Models when the Retailer Does Not Adopt the Recommender System

In practice, not all retailers opt to implement recommender systems. To gain a more comprehensive understanding of the manufacturer's strategies regarding the adoption of recommender system, it is essential to analyze the strategic choices when the retailer does not utilize such system. This analysis will facilitate a deeper comprehension of the manufacturer's optimal strategies across various scenarios and shed light on the implications of these strategies for both the market and the supply chain.

In this section, specifically when the retailer does not adopt the recommender system, we analyze three scenarios: (i) the manufacturer also refrains from using the recommender system in the direct-sale channel, referred to as scenario "NN"; (ii) the manufacturer adopts the recommender system in the direct-sale channel under the CPS payment scheme, referred to as scenario "NS"; and (iii) the manufacturer adopts the recommender system in the direct-sale channel under the CPC payment scheme, referred to as scenario "NC".

4.1 Scenario NN: No Firm Adopts the Recommender System

In scenario NN, similar to Section 3, for the traditional market, the demands in the direct-sale and wholesale channels are

$$D_m = \frac{(1-\theta)\alpha - p_m + \theta p_r}{1-\theta^2} \text{ and } D_r = \frac{(1-\theta)\alpha - p_r + \theta p_m}{1-\theta^2}.$$

We compute the manufacturer's and retailer's profits as $\pi_m = wD_r + p_m D_m$ and $\pi_r = (p_r - w)D_r$, respectively. In the game, the manufacturer first determines and announces both p_m and w, and the retailer then determines p_r.

One can solve the "leader-follower" game to find that the manufacturer's and the retailer's prices in Stackelberg equilibrium are $p_m^{NN} = w^{NN} = \alpha/2$ and $p_r^{NN} = \alpha(3-\theta)/4$. The two firms' profits are $\pi_m^{NN} = (3+\theta)\alpha^2/[8(1+\theta)]$ and $\pi_r^{NN} = (1-\theta)\alpha^2/[16(1+\theta)]$. When the channel competition is more intensive (i.e., the value of θ increases), the retailer reduces the retail price to increase the demand. As a result, the demand in the manufacturer's direct channel decreases, which makes the manufacturer worse off. That is, the double marginalization is harmful to all members in the supply chain.

4.2 Scenario NS: Only the Manufacturer Adopts the Recommender System under the CPS Payment Scheme

The retailer does not adopt the recommender system in the wholesale channel but the manufacturer uses the service of the recommender system who chooses the CPS payment scheme in the direct-sale channel. We can easily derive the demand in the recommended market as $\hat{D}_m = \hat{\alpha} - p_m$. The manufacturer's, retailer's, and recommender system's profit functions are $\pi_m = wD_r + p_m(D_m + \hat{D}_m) - p_e f(\hat{D}_m)$, $\pi_r = (p_r - w)D_r$, and $\pi_e = (p_e - c)f(\hat{D}_m)$. We can find the prices in Stackelberg equilibrium for scenario NS as

$$\begin{aligned} p_m^{NS} &= \frac{c + 5\hat{\alpha} + \alpha}{8},\ w^{NS} = \frac{\alpha(4 - 3\theta) + (c + 5\hat{\alpha})\theta}{8}; \\ p_r^{NS} &= \frac{\alpha(6 - 5\theta) + (c + 5\hat{\alpha})\theta}{8},\ \text{and } p_e^{NS} = \frac{c + 3\hat{\alpha} - \alpha}{2}. \end{aligned}$$

Using equilibrium prices, we compute the three firms' profits as

$$\begin{aligned} \pi_m^{NS} &= \frac{\alpha^2(5 - 3\theta) + c^2(1 + \theta) - 23\hat{\alpha}^2(1 + \theta) - 2c(3\hat{\alpha} - \alpha)(1 + \theta) + 26\hat{\alpha}\alpha(1 + \theta)}{32(1 + \theta)}, \\ \pi_r^{NS} &= \frac{\alpha^2(1 - \theta)}{16(1 + \theta)},\ \text{and } \pi_e^{NS} = \frac{(c - 3\hat{\alpha} + \alpha)^2}{16}. \end{aligned}$$

To ensure that the equilibrium outcomes are non-negative, we consider recommendation cost c such that $c \in \left(c_l^{NS}, c_h^{NS}\right)$, where $c_l^{NS} \equiv \max\{\alpha - 3\hat{\alpha}, 0\}$ and $c_h^{NS} \equiv \min\{\alpha(7 + 5\theta)/(1 + \theta) - 5\hat{\alpha}, 3\hat{\alpha} - \alpha, 1\}$.

Proposition 3 *For scenario NS, p_e^{NS}, p_m^{NS}, w^{NS}, and p_r^{NS} are all increasing in $\hat{\alpha}$ and c.*

Proposition 3 discloses that as recommendation strength and unit recommendation cost increase, the recommender system raises the recommendation service price. The result reflects the increased value attributed to the recommendations provided. Consequently, the manufacturer, in response to the higher recommendation service price, adjusts both the direct-channel retail price and the wholesale price. This strategic operation aims to maintain profitability while accommodating the elevated cost of recommendation service. Ultimately, these adjustments cascade down to the retailer, compelling them to align their retail price with the changes observed in the supply chain. This intricate interplay underscores the dynamic nature of pricing decisions in the context of the adoption of recommender system, highlighting the interactions and strategic pricing responses observed across the supply chain.

Proposition 4 *For scenario NS, we find the following two results. (i) if $c \in \left(0, c_1^{NS}\right) \cap \left(c_l^{NS}, c_h^{NS}\right)$, π_m^{NS} increases with $\hat{\alpha}$; otherwise, π_m^{NS} decreases with $\hat{\alpha}$. In addition, π_r^{NS} is independent of $\hat{\alpha}$, and π_e^{NS} always increases with $\hat{\alpha}$. (ii) π_m^{NS} and π_e^{NS} decrease with c, whereas π_r^{NS} is independent of c.*

From Proposition 4(i), we note that the manufacturer's profit increases with the recommendation strength when the unit recommendation cost is sufficiently small. This suggests that under such conditions, a higher recommendation strength can yield greater benefits for the manufacturer. However, if the unit recommendation cost is sufficiently large, the recommender system's service price increases with the recommendation strength, posing a cost burden for the manufacturer. Consequently, an increase in the recommendation strength may result in a decline in the manufacturer's profit. In addition, as the retailer does not adopt any recommender system, the recommendation strength does not affect their operations. Nonetheless, the recommender system stands to gain from a higher recommendation strength, as it can attract more consumers to make purchases through online recommendations, thereby increasing its profitability. Proposition 4(ii) further demonstrates that due to the increase in the recommendation cost and the rise in the recommendation service price, the manufacturer opts to set a higher direct-channel retail price. However, a significant increase in retail price may lead to a considerable drop in demand, possibly causing the retailer to experience a decline in his profit as the recommendation strength and cost increase.

4.3 Scenario NC: Only the Manufacturer Adopts the Recommender System under the CPC Payment Scheme

The three firms' profit functions are $\pi_m = wD_r + p_m(D_m + \hat{D}_m) - p_e f(\hat{D}_m)$, $\pi_r = (p_r - w)D_r$, and $\pi_e = (p_e - c)f(\hat{D}_m)$. We compute the prices in Stackelberg equilibrium as

$$\begin{aligned} p_m^{NC} &= \frac{c + k\hat{\alpha}(5\hat{\alpha} + \alpha)}{8k\hat{\alpha}},\ w^{NC} = \frac{c\theta + k\hat{\alpha}[\alpha(4 - 3\theta) + 5\hat{\alpha}\theta]}{8k\hat{\alpha}}; \\ p_r^{NC} &= \frac{c\theta + k\hat{\alpha}[\alpha(6 - 5\theta) + 5\hat{\alpha}\theta]}{8k\hat{\alpha}},\ \text{and}\ p_e^{NC} = \frac{c + k\hat{\alpha}(3\hat{\alpha} - \alpha)}{2}. \end{aligned}$$

The three firms' profits are

$$\begin{aligned} \pi_m^{NC} &= \frac{c^2(1+\theta) - 2kc\hat{\alpha}(3\hat{\alpha} - \alpha)(1+\theta) + k^2\hat{\alpha}^2[\alpha^2(5 - 3\theta) - 23\hat{\alpha}^2(1+\theta) + 26\hat{\alpha}\alpha(1+\theta)]}{32k^2\hat{\alpha}^2(1+\theta)}, \\ \pi_r^{NC} &= \frac{\alpha^2(1-\theta)}{16(1+\theta)},\ \text{and}\ \pi_e^{NC} = \frac{[c + k\hat{\alpha}(-3\hat{\alpha} + \alpha)]^2}{16k^2\hat{\alpha}^2}. \end{aligned}$$

For non-negative results, we require $c \in (c_l^{NC}, c_h^{NC})$ where $c_l^{NC} \equiv \max\{k\hat{\alpha}(\alpha - 3\hat{\alpha}), 0\}$ and $c_h^{NC} \equiv \min\{k\hat{\alpha}[\alpha(7+5\theta)/(1+\theta) - 5\hat{\alpha}], k\hat{\alpha}(3\hat{\alpha} - \alpha), 1\}$.

Proposition 5 *For scenario NC, p_m^{NC}, w^{NC}, and p_r^{NC} increase with $\hat{\alpha}$ and c. In addition, p_e^{NC} increases with c, and p_e^{NC} increases with $\hat{\alpha}$ if $\hat{\alpha} > \alpha/6$ but decreases with $\hat{\alpha}$ otherwise.*

We find from Proposition 6 that the impacts of recommendation strength and unit recommendation cost on the manufacturer's direct-channel retail price, wholesale price and the retailer's retail price are identical to those in Proposition 3. However, the recommendation service price exhibits non-monotonic behavior with respect to recommendation cost. Specifically, when the value of recommendation strength is sufficiently low, the recommendation service price decreases with the recommendation strength but increases otherwise. This result can be explained as follows: when the recommendation strength is sufficiently low, the recommender system fails to significantly boost the manufacturer's sales, prompting the recommender system to lower the service price in order to incentivize the manufacturer's adoption of its recommendation service. Conversely, as the recommendation strength reaches a certain threshold, the recommender system gradually raises the service price.

Proposition 6 *For scenario NC, we obtain the following two results. (i) If $\hat{\alpha} < \hat{\alpha}^{NC}$ and $c \in (0, c_1^{NC}) \cap (c_l^{NC}, c_h^{NC})$, then π_m^{NC} increases with $\hat{\alpha}$; otherwise, π_m^{NC} decreases with $\hat{\alpha}$. In addition, π_r^{NC} is independent of $\hat{\alpha}$, and π_e^{NC} always increases with $\hat{\alpha}$. (ii) both π_m^{NC} and π_e^{NC} decrease with c, and π_r^{NC} is independent of c.*

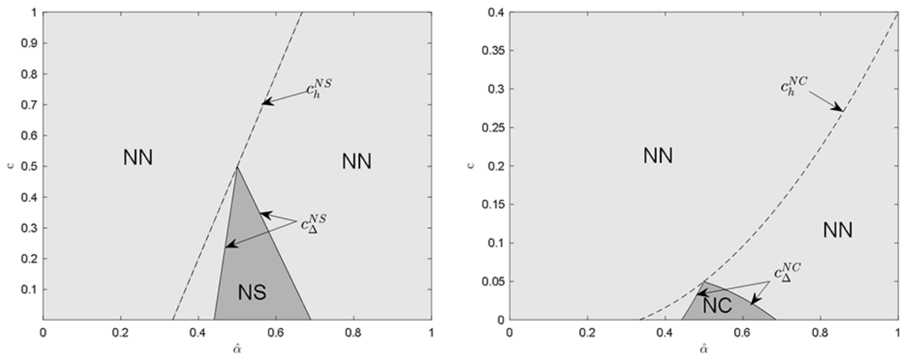
The findings presented in Proposition 6 are similar to those in Proposition 4. However, it is intriguing to note that the manufacturer's profit exhibits an upward trend with the recommendation strength when both recommendation cost and recommendation strength are sufficiently low. Specifically, in the scenarios where both the recommendation strength and recommendation cost are minimal, an increase in recommendation strength correlates positively with the manufacturer's profit. This indicates that when the system operates efficiently and costs are manageable, a higher recommendation strength can increase the sales and also the manufacturer's profit. However, regardless of whether the recommendation strength or recommendation cost is higher, any escalation in the recommendation service price, prompted by an increase in recommendation strength, tends to reduce the manufacturer's profit. This exposes the importance of considering both the recommendation strength and cost in maximizing the manufacturer's profitability in the context of the recommender system adoption. Therefore, although an increase in the recommendation strength could benefit the manufacturer, it is contingent upon the recommendation strength and recommendation cost that is sufficiently low.

4.4 The Manufacturer's Strategic Choice when the Retailer Does Not Adopt the Recommender System

To explore whether it is beneficial for the manufacturer to adopt the recommender system under the CPS and CPC payment schemes, we compare the manufacturer's profits for scenarios NS and NN as well as those for scenarios NC and NN.

Theorem 4 *If the retailer does not adopt the recommender system, we find that, regardless of whether the recommender system chooses the CPS or CPC payment scheme, the manufacturer should choose the recommender system if $c < c_\Delta^{NS}$ ($c < c_\Delta^{NC}$) but should not do it otherwise.*

We plot Figure 2 to visually illustrate the findings in Theorem 4. When the unit recommendation cost exceeds a certain threshold, the recommender system is motivated to set a notably high service price, as such decision helps shift the burden of recommendation cost to the manufacturer who uses the recommendation services. Consequently, faced with the prospect of exorbitant service fees, the manufacturer may opt against adopting the recommender system. Moreover, the manufacturer's decision regarding the adoption of recommender system is also influenced by the recommendation strength. When the recommendation strength increases, a greater number of consumers are likely to buy according to recommendations. As a result, the manufacturer becomes more likely to embrace the recommender system. However, if the recommendation strength reaches a sufficiently high level, the manufacturer is confronted with the prospect of paying a substantial recommendation service price. This may erode any incentive for the manufacturer to adopt the recommender system, although the manufacturer may benefit by achieving higher sales from a higher recommendation strength.



(a) CPS (b) CPC

Figure 2: The manufacturer's strategic choices when the retailer does not adopt recommender system ($\alpha = 1$, $\theta = 0.8$, $k = 0.2$)

4.5 The Manufacturer's Preference when the Retailer Does Not Adopt the Recommender System

We investigate whether the manufacturer prefers to choose the CPS or CPC payment scheme, when the manufacturer uses the recommender system but the retailer does not. To this end, we compare the manufacturer's profits in scenarios NS and NC.

Theorem 5 *When the manufacturer adopts the recommender system and the retailer does not, the manufacturer prefers the CPS payment scheme to the CPC payment scheme, i.e.,* $\pi_m^{NS} > \pi_m^{NC}$.

Theorem 5 indicates that, in scenarios where only the manufacturer decides to employ a recommender system, the manufacturer always collaborates with a recommender system that uses the CPS payment scheme. Our justification is as follows: in the online marketplace, consumers who click on product links and browse product details may not necessarily make a purchase. This inherent uncertainty surrounding online sales implies that, under the CPC payment scheme, the manufacturer may incur unnecessary costs. Furthermore, as found previously, the recommendation strength is typically not negligible. Consequently, a considerable number of consumers are likely to be aware of the recommender system and engage by clicking on product links to make purchases. This makes the CPS payment scheme as the preferable option.

Moreover, the CPC payment scheme may inadvertently induce more fraud-click behaviors, which refer to those clicks that a person or a firm, in order to obtain commercial profits, use automated scripts and computer programs or employ labor force to imitate legitimate network users and increase the expenditure of the income of publishers. For example, Baidu, an internet company in China sued the Woai Network Technology Company because the defendant helped users create false click data, and its misconduct breached the anti-unfairness competition law. Consequently, the manufacturer would be wise to opt for a payment scheme under which costs are associated solely with actual sales volume, thereby mitigating the risk of fraudulent click activities.

5 Win-win Situation Analysis for the Adoption of Recommendation Services

In the preceding two sections, we examined the manufacturer's strategies when the retailer either adopts or does not adopt the recommender system. Nevertheless, these analyses were conducted from a unilateral perspective and did not adequately account for the collaborative dynamics between the manufacturer and the retailer. This section aims to investigate the win-win scenarios regarding the retailer's and manufacturer's preferences for adopting recommendation services under CPS and CPC payment schemes.

5.1 Win-win Situation Analysis when the Recommender System Uses the CPS Payment Scheme

When the recommender system uses the CPS payment scheme, there are four possible scenarios, i.e., (N,N), (N,S), (S,N), and (S,S), where the first letter represents the retailer's strategy and the second letter represents the manufacturer's strategy. "N" and "S" denote not using and using the CPS recommender system, respectively. Then we can derive the following two win-win situations: (N,N) and (N,S) in proposition 7. Specifically, if the recommendation strength is sufficiently small or when it is moderate but the recommendation cost is sufficiently small, the win-win situation is (N,N). Otherwise, if the recommendation strength is moderate but the recommendation cost is sufficiently large, the win-win situation is (N,S).

Proposition 7 *Scenaio NN is the a win-win situation when* $c \in (\tilde{c}_{r1}^{NN}, \tilde{c}_{r2}^{NN}) \cap (\tilde{c}_{m1}^{NN}, \tilde{c}_{m2}^{NN})$ *and Scenaio NS is a win-win situation when* $c \in (\tilde{c}_{r1}^{NS}, \tilde{c}_{r2}^{NS}) \cap \{(0, \tilde{c}_{m1}^{NN}) \cup (\tilde{c}_{m2}^{NN}, 1)\}$.

We visually show the win-win situations of the manufacturer and retailer in Figure 3, in which there are five zones each representing a region defined with respect to the values of recommendation strength $\hat{\alpha}$ and cost c. The win-win situations (N,N) and (N,S) are located in Zones I and IV, as shown in the shaded part of the figure.

We learn from Figure 3 that, when the recommendation strength is sufficiently low (Zone I), the utilization of the recommendation service fails to significantly bolster sales growth for both the manufacturer and the retailer. Consequently, neither the manufacturer nor the retailer has an incentive to adopt the recommender system. When the recommendation strength is moderately low, the retailer exhibits a preference for scenario SN (corresponding to Zones II and III). Actually, the higher recommendation strength is advantageous for the retailer, particularly when the manufacturer refrains

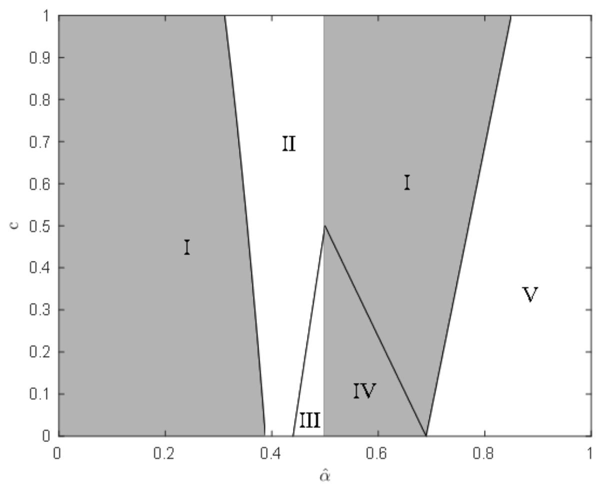


Figure 3: Win-win situations for the manufacturer and the retailer under the CPS payment scheme ($\alpha = 1, \theta = 0.8, k = 0.2$).

from adopting the recommender system. There are two possible cases regarding the manufacturer's recommendation preference. In one case, the recommendation cost is sufficiently large, which corresponds to Zone II. For this case, the manufacturer prefers to sell products with no recommendation service. In the other case, the recommendation cost is sufficiently low, which corresponds to Zone III. The manufacturer prefers to adopt the recommender system in his direct sale channel, anticipating the retailer's reluctance to adopt the recommendation services in the wholesale channel. Such result is justified by the manufacturer's prioritization of efforts towards enhancing efficiency in his direct sales channel to minimize double marginalization and enhance supply chain efficiency. The interests of the retailer and manufacturer are in complete conflict in Zone III. By comparing Zones II and III, one can find that the retailer can undertake a greater transfer cost from the recommender system than the manufacturer.

In Zone I and IV, as the recommendation strength increases to a moderately high level, the benefits resulting from by the recommendation service are reduced by higher transfer costs borne by both parties. When the recommendation cost is relatively high (as in Zone I), the marginal benefits derived from the recommendation service pale in comparison to the transfer payments necessitated by the firms. Consequently, a win-win situation is reached wherein neither party expects to adopt the recommender system. However, when the recommendation cost is relatively low (in Zone IV), the manufacturer and the retailer achieve the win-win situation in which only the manufacturer adopts the recommender system. That is, the manufacturer who can afford a low transfer recommendation cost prefers to adopt the recommender system by himself rather than together with the retailer. This strategic choice allows the manufacturer to mitigate the double marginalization issues, as well as to avoid free-riding concerns from the retailer. In Zone V, marked by a high recommendation strength, there are no win-win situations of the firms' recommendation strategies. The retailer favors a scenario where both firms adopt the recommender system, while the manufacturer prefers a scenario where neither does.

5.2 Win-win Situation Analysis when the Recommender System Uses the CPC Payment Scheme

When the recommender system uses the CPC payment scheme, similar to 5.1, there are four possible scenarios: (N,N), (N,C), (C,N), and (C,C). Then we can find that (N,N) and (N,C) are two win-win situations as shown in proposition 8. Specifically, when the recommendation strength is sufficiently low or when the recommendation cost is sufficiently high, both the manufacturer and the retailer prefer to choose strategy "N," and (N,N) is a win-win situation. When the recommendation strength is moderate and the recommendation cost is sufficiently low, the manufacturer and the retailer prefer to choose strategy "N" and "C," respectively, and (N,C) is a win-win situation.

Proposition 8 *Scenaio NN is the win-win situation when* $c \in (\tilde{c}_{r3}^{NN}, \tilde{c}_{r4}^{NN}) \cap (\tilde{c}_{m3}^{NN}, \tilde{c}_{m4}^{NN})$ *and Scenaio NC is the win-win situation when* $c \in (\tilde{c}_{r1}^{NC}, \tilde{c}_{r2}^{NC}) \cap \{(0, \tilde{c}_{m3}^{NN}) \cup (\tilde{c}_{m4}^{NN}, 1)\}$.

We present Figure 4 to show four possible regions determined by the recommendation strength and cost, where the shaded area represents the win-win situation between the retailer and the manufacturer. Our analysis reveals that the win-win situations under the CPC payment scheme are similar to those under the CPS payment scheme. For a sufficiently low recommendation strength or a sufficiently high recommendation cost (which corresponds to Zone I), the effect of recommendation service on driving substantial sales growth for the manufacturer and the retailer is limited, thereby leading neither of them to have a sufficient incentive to adopt the recommender system. When the recommendation cost is sufficiently low (corresponding to Zones II and III), the manufacturer desires to adopt the recommender system in his direct sales channel but does not expect the retailer to do so in the wholesale channel. If the recommendation strength is moderately low (as in Zone II), the retailer also desires to use the recommendation service at a relatively low price to promote sales.

However, when the recommendation strength is moderately high (as in Zone III), the retailer is better off from abandonment of the recommender system. Therefore, the retailer avoids not only the possible free-riding by the manufacturer but also the additional costs associated with a low click-through rate. That is, the optimal choice for the retailer is to allow the manufacturer to adopt the recommender system, which corresponds to a win-win situation. For a sufficiently high recommendation strength, the two firms can not achieve a win-win situation. The retailer favors the scenario in which both firms adopt the recommender system, whereas the manufacturer prefers a scenario in which no firm chooses the system.

Comparing the results for the CPS and CPC payment schemes, we find that under the CPC scheme, the win-win situation for the case of only the manufacturer adopting the recommender system is smaller than that under the CPS scheme. However, the area where no firm adopts the recommender system is substantially larger. This is due to the fact that under the CPC payment scheme, such adoption fails to yield improvements in sales. In addition, the recommendation cost plays an important role in influencing the decisions of both the manufacturer and the retailer. Specifically, when the recommendation cost is sufficiently low, both firms may opt to adopt the recommendation service. Conversely, when the recommendation cost exceeds a certain threshold, the firms cannot achieve any win-win situation.

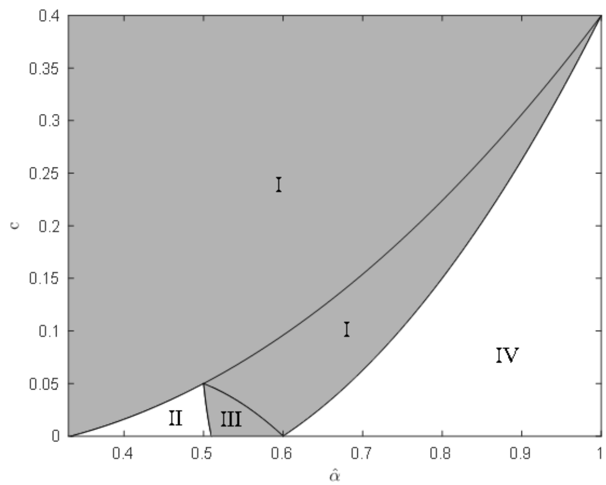


Figure 4: Win-win situations for the manufacturer and the retailer under the CPC payment scheme ($\alpha = 1, \theta = 0.8, k = 0.2$).

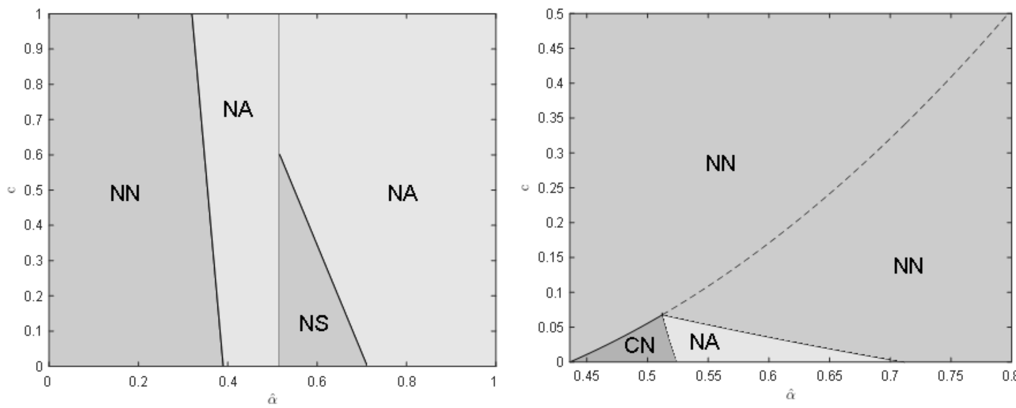
6 Extensions

In this section, by relaxing certain assumptions or modifying specific settings, we extend our basic model to the following four scenarios: First, direct and retail prices are determined simultaneously. Second, we incorporate consumers' outside options into the analysis. Third, we account for heterogeneous consumer preferences across channels. Fourth, we examine the coexistence of CPS and CPC payment schemes. Through analyzing these four cases, we aim to evaluate the robustness of our primary findings and offer additional insights into the role and impact of recommender systems in diverse contexts.

6.1 Simultaneous Pricing Decisions

In this extension, we consider a model for simultaneous decision-making on direct and retail prices. Under CPS and CPC recommendation payment schemes, the sequence of pricing is as follows. The recommender system first determines its service price. Secondly, the manufacturer determines the wholesale prcie w. Thirdly, the retailer and manufacturer make decisions on p_r and p_m simultaneously. As with the basic model, we obtain the equilibrium results by backward induction. Based on this, we analyze the win-win situations of the manufacturer and the retailer, which is solved similarly to Section 5. The specific results are as follows.

The win-win situations of the retailer and manufacturer are shown in Figure 5 (NN, NS and CN denote the win-win region and NA denotes that the region is not a win-win situation. We still use parameter values $\alpha = 1$, $\theta = 0.8$, $k = 0.2$). First, through model comparison, we find that the win-win situations of the retailer and manufacturer under the CPS payment scheme are consistent with the benchmark model. Under the CPS payment scheme, when the recommendation strength is low and recommendation cost is large, consistent with the case where the manufacturer has priority pricing, the retailer and the manufacturer reach a win-win situation in which neither adopts the recommender



(a) CPS (b) CPC

Figure 5: Win-win situations of the manufacturer and retailer under the simultaneous pricing decisions.

system. However, when the recommendation strength is high, both firms prefer not to adopt the recommender system. When the recommendation strength and recommendation cost are both low, the two firms reach a win-win situation in the CN model, which is the opposite of the benchmark model. This result suggests that the power structure has an impact on both recommender system usage strategies under the CPC payment method. When the manufacturer has an absolute power advantage, it has the ability to pay for recommendation. And when the manufacturer does not have an absolute power advantage, he prefers to obtain recommendations indirectly through the retailer's adoption of the recommender system.

6.2 Outside Competition

In our base model, it is assumed that consumers will always purchase products from either the manufacturer or retailer. In the extension of this subsection, we relax this assumption, i.e., we allow consumers to be able to not purchase products from the manufacturer and retailer. That is to say, in a highly competitive market, consumers can make purchases from other manufacturers as well as not buy the product. We characterize this option of outside competition as d_o. The greater the outside competition, the lower the utility consumers receive from purchasing the manufacturer's or retailer's product. The consumer utility function can be expressed as:

$$U = \sum\nolimits_{i=m,r} \left[(\alpha - d_o) D_i - p_i D_i - \frac{1}{2} D_i^2 \right] - \theta D_m D_r.$$

Maximizing U, we have the aggregate demands in direct-sale and wholesale channels of the traditional market as

$$D_m = \frac{(1-\theta)(\alpha - d_o) - p_m + \theta p_r}{1-\theta^2} \text{ and } D_r = \frac{(1-\theta)(\alpha - d_o) - p_r + \theta p_m}{1-\theta^2}.$$

Obviously, from the demand function, the greater the outside competition, the lower the market demand in the direct and wholesale channels.

Similarly, when either the manufacturer or the retailer adopts the recommender system, the demand in direct-sale or wholesale channel of the recommended market is $\hat{D}_i = \hat{\alpha} - d_o - p_i$, $i = m, r$. And if both adopt the recommender system, the aggregate demands in direct-sale and wholesale channels of the recommended market are $\hat{D}_m = [(1-\theta)(\hat{\alpha} - d_o) - p_m + \theta p_r] / \left(1 - \theta^2\right)$ and $\hat{D}_r = [(1-\theta)(\hat{\alpha} - d_o) - p_r + \theta p_m] / \left(1 - \theta^2\right)$. Then by backward induction, we can derive the equilibrium results under each model. The equilibrium outcomes in each model are shown in Appendix.

Without loss of generality, we analyze the sensitivity analysis in the Scenario CC only here. With respect to the effect of outside competition d_o on each party's profits $\bar{\pi}_i^{CC}$ ($i \in \{m, r, e\}$), we obtain the following corollary.

Corollary 1 *$\bar{\pi}_m^{CC}$ decreases with d_o when $d_o < d_{om}$, and increases with d_o when $d_o > d_{om}$. $\bar{\pi}_r^{CC}$ increases with d_o when $d_o < d_{or}$, and decreases with d_o when $d_o > d_{or}$. $\bar{\pi}_e^{CC}$ decreases with d_o when $d_o < d_{oe}$, and increases with d_o when $d_o > d_{oe}$.*

In a market with less competition, a manufacturer may be able to achieve higher profits through higher prices. However, as outside competition d_o increases, the manufacturer may need to lower his price, resulting in lower profit. For the retailer, as outside competition increases, he may be able to increase sales and profits by offering more competitive prices to attract customers. In a highly competitive market, the manufacturer may rely more on recommender systems to increase the visibility and attractiveness of products, and thus may be willing to pay more to recommender systems in order to maintain or increase market share. At this stage, the recommender system is more inclined to collaborate with the market leader, namely the manufacturer, thereby enhancing profitability. In contrast, the retailer encounters greater pricing pressure and must reduce prices to attract customers, which leads to diminished profits. Simultaneously, if the retailer does not heavily depend on the recommender system, they may fail to fully leverage the benefits provided by the system, consequently impacting their profitability.

Further analysis shows that when considering the option of external competition, the win-win situation is simply a translation on the base model and does not affect the main nature of the conclusions obtained. Therefore, the main results of this study still hold in this case, which means that our results are robust and our main model is an appropriate and acceptable simplification.

6.3 Consumer Heterogeneity in Channel Preferences

In our previous analyses, the model was deterministic, i.e., all consumers received the same utility for the purchased product. However, in reality different consumers may have different preferences for the same product or purchase channel. In order to make the model more general and close to reality, we consider the heterogeneity of consumers' preferences for different channels. With reference to Zhou and Zou (2023), we denote t_j as the preference of consumer j between channels and it is uniformly distributed between $[0, t]$ (when $t > 0$) or $[t, 0]$ (when $t < 0$), where t denotes the level of preference heterogeneity across consumers. Here, we assume that $t \in [-1, 1]$, which $t \in [0, 1]$ means that consumers may prefer the direct-sale channel and $t \in [-1, 0)$ means that consumers may prefer the wholesale channel. Therefore, the utility function can be rewritten as:

$$U_j = (\alpha + t_j)D_m + (\alpha - t_j)D_r - p_m D_m - p_r D_r - \frac{1}{2}D_m^2 - \frac{1}{2}D_r^2 - \theta D_m D_r.$$

Maximizing U_j, we can get the demands in direct-sale and wholesale channels as

$$D_m = \frac{(1-\theta)\alpha - p_m + \theta p_r}{1-\theta^2} + \frac{t_j}{1-\theta} \text{ and } D_r = \frac{(1-\theta)\alpha - p_r + \theta p_m}{1-\theta^2} - \frac{t_j}{1-\theta}.$$

Accordingly,

$$E[D_m] = \frac{(1-\theta)\alpha - p_m + \theta p_r}{1-\theta^2} + \frac{t}{2(1-\theta)} \text{ and } E[D_r] = \frac{(1-\theta)\alpha - p_r + \theta p_m}{1-\theta^2} - \frac{t}{2(1-\theta)}.$$

By backward induction, we find the equilibrium results under each model. Here we provide a specific analysis for scenario SS, and the other scenarios are similar.

Corollary 2 *$\breve{\pi}_m^{SS}$ decreases with t when $t \in [-1, t_m)$, and increases with t when $t \in [t_m, 1]$. $\breve{\pi}_r^{SS}$ decreases with t when $t \in [-1, t_r)$, and increases with t when $t \in [t_r, 1]$. $\breve{\pi}_e^{SS}$ decreases with t when $t \in [-1, t_e)$, and increases with t when $t \in [t_e, 1]$.*

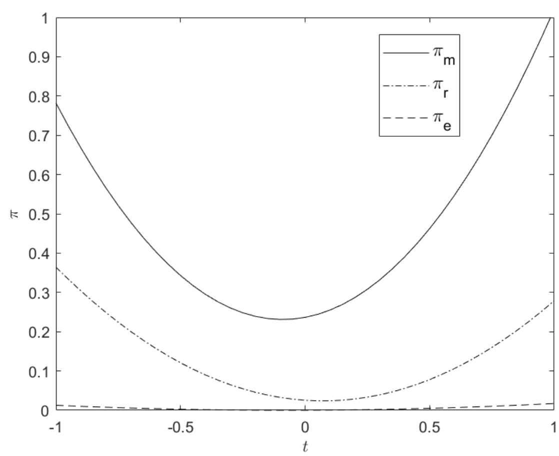


Figure 6: The impact of t on the manufacturer, retailer and recommender system's profits.

Corollary 2 and Figure 6 ($\alpha = 1$, $\theta = 0.8$, $c = 0.1$, $\hat{\alpha} = 0.4$) show that when the level of consumer heterogeneity takes values from -1 to 1, the profits of the manufacturer, the retailer and the recommender system show a U-shape regarding consumer heterogeneity. This is because when the level of consumer heterogeneity is low (close to 0), i.e., most consumers have very similar preferences, direct and wholesale channels may offer similar products or services, leading to homogeneous competition. This competition may give rise to price wars and compression of profit margins. When the level of heterogeneity begins to increase, consumer needs and preferences begin to start to diversify. This provides opportunities for the manufacturer and retailer to increase profits by satisfying the needs of different groups of consumers through different distribution channels. Accordingly, the profitability of recommender systems increases. This extension does not affect the qualitative results of the original model in equilibrium, but enriches the conclusions of the original model.

6.4 Co-existence of CPS and CPC Payment Schemes

In reality, there may be multiple payment schemes for recommender systems, or one recommender system may offer multiple payment schemes at the same time. In this subsection, we consider the possibility that a recommender system can offer both CPS and CPC payment schemes, and to explore whether the manufacturer and retailer have incentives to choose another payment scheme when both of them use the recommender system. By solving the model, we have the following results.

Corollary 3 *(i) If the retailer adopts CPS payment scheme, the manufacturer prefers CPS when* $\phi_1 < c < \phi_2$*, and prefers CPC otherwise. (ii) When* $\sqrt{3}-1 < \theta \leq \left[\sqrt{2\hat{\alpha}^2+4\hat{\alpha}+3}-1\right]/(\hat{\alpha}+1)$ *and* $0 < k < \left(-\theta^2-2\theta+2\right)/\left(\theta^2\hat{\alpha}-2\hat{\alpha}\right)$*, or* $\left[\sqrt{2\hat{\alpha}^2+4\hat{\alpha}+3}-1\right]/(\hat{\alpha}+1) < \theta < 1$*, if the retailer adopts CPC payment scheme, the manufacturer prefers CPS when* $c > \phi_3$*, and prefers CPC otherwise. (iii) If the manufacturer adopts CPS payment scheme, the retailer prefers CPC when* $c > \phi_4$*, and prefers CPS otherwise. (iv) When* $1/2 < \theta \leq (1+\hat{\alpha})/2$ *and* $0 < k < (2\theta-1)/\hat{\alpha}$*, or* $\theta > (1+\hat{\alpha})/2$*, if the manufacturer adopts CPC payment scheme, the retailer prefers CPC when* $\phi_5 < c < \phi_6$*, and prefers CPS otherwise.*

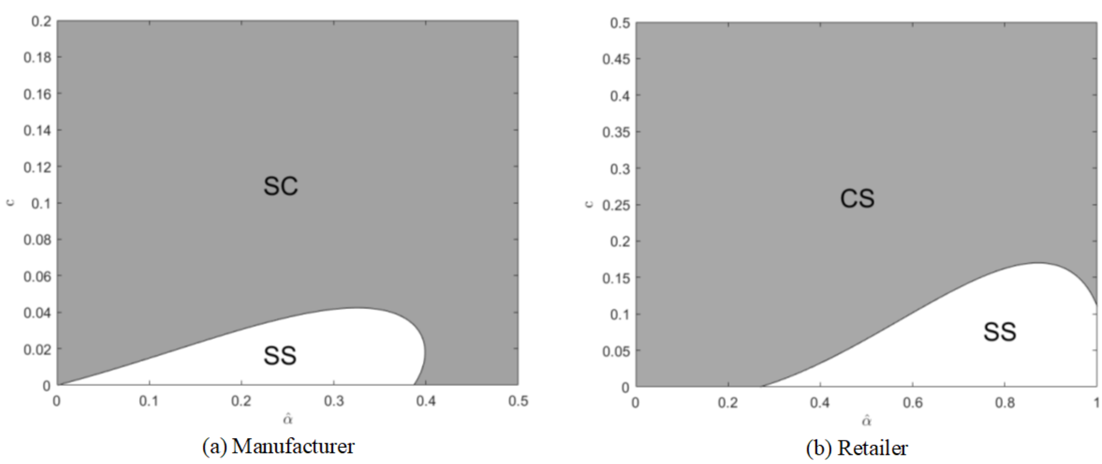
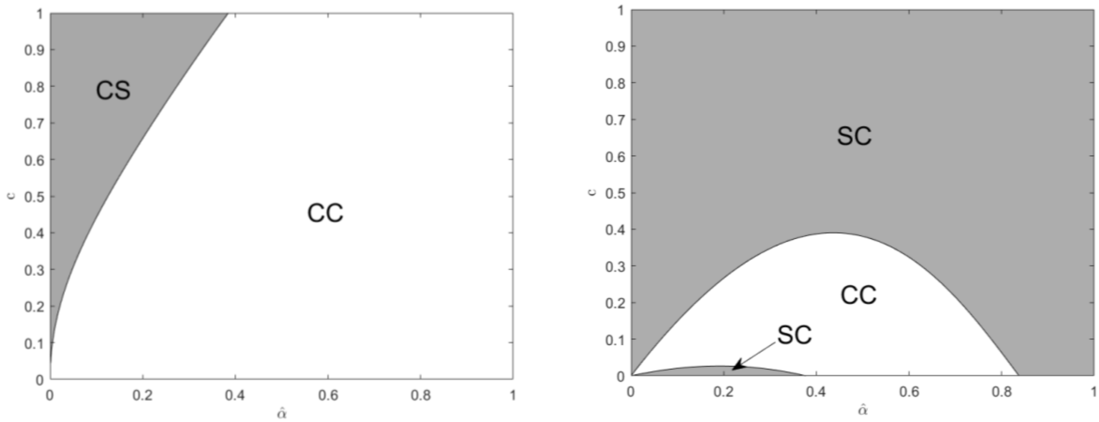


Figure 7: The manufacturer and retailer's preferences of payment scheme when competitors choose CPS payment scheme.

From Corollary 3, Figures 7 and 8 ($\alpha = 1$, $\theta = 0.8$, $k = 0.2$), we can see that under the CPS payment scheme, the manufacturer and retailer prefer the CPC payment scheme when the recommendation cost is high. The CPC payment scheme incentivizes the manufacturer and retailer to attract more traffic, despite the potentially high recommendation costs. By increasing the number of clicks, they can enhance product exposure and capitalize on potential sales opportunities. When both firms adopts CPC recommender systems, for the manufacturer, he is willing to move from CPC to CPS



(a) Manufacturer (b) Retailer

Figure 8: The manufacturer and retailer's preference of payment scheme when competitors choose CPC payment scheme.

recommender system only when the recommendation strength is low and the recommendation cost is high. With low recommendation strength and high recommendation cost, the clicks a manufacturer receives through the CPC payment scheme do not convert into enough sales, resulting in sales inefficiencies. Moving to CPS payment scheme reduces this inefficiency because the manufacturer only has to pay when an actual sale occurs. For retailers who are already at a disadvantage in the game due to costly recommendations, adopting the recommender system with CPS payment scheme ensures that they only incur costs when an actual sale occurs, thereby effectively reducing expenses.

7 Summary and Concluding Remarks

We investigate the impact of a recommender system on a supply chain in which a manufacturer and a retailer operates under a dual-channel competition. The manufacturer distributes its products through both a direct channel and a wholesale channel involving the retailer. We investigate whether the manufacturer should adopt the recommendation service under two scenarios: when the retailer adopts the recommender system and when it does not. Specifically, when the retailer does or does not adopt the recommender system that uses the CPS or CPC payment scheme, we explore whether the manufacturer has an incentive to adopt the system or not.

We derive the manufacturer's and the retailer's optimal pricing decisions in Stackelberg game, perform analytic sensitivity analyses, and draw a number of managerial implications. When the retailer does not adopt the recommender system, the manufacturer's recommendation strategy hinges on the recommendation strength and cost. Regardless of whether the recommender system uses the CPS or CPC payment scheme, we find that if the recommendation cost is sufficiently high, the manufacturer is better off from not choosing the recommender system. Otherwise, the manufacturer's decision to adopt the recommender system depends on the recommendation strength. A sufficiently low recommendation strength does not prompt the manufacturer to adopt the recommender system. However, as the recommendation strength increases, more consumers make purchases via recommendations, leading to higher sales and profits, thereby incentivizing the manufacturer to adopt the recommender system. However, for a sufficiently high recommendation service price and significant strength, the manufacturer may opt to forego the recommender system.

When the retailer adopts the recommender system, regardless of whether the recommender system employs the CPS or CPC payment scheme, the manufacturer's optimal strategy is not to adopt the recommender system. This occurs because there is a duplicated cost associated with the recommendation adoption. Since the retailer uses recommendations in the wholesale channel, it may not be efficient for the manufacturer to do so in the direct-sale channel. The manufacturer can take a "free ride" by the retailer's product recommendation in the wholesale channel, benefiting without incurring additional costs. Moreover, when the retailer does not adopt the recommender system in the wholesale channel, the channel has a less influence on consumers. If the manufacturer adopts the recommender system in the direct-sale channel, he can more effectively influence consumers' choices and purchasing decisions, thereby increasing sales in the direct-sale channel. However, the retailer's adoption of recommendation in the wholesale channel can enhance the influence of the wholesale channel on consumers. If the manufacturer also adopts the recommender system in the direct-sale channel, it may compete with the wholesale channel to some extent, which is not conducive to the maintaining the channel relationship.

Regarding the impact of payment schemes on the manufacturer's recommendation strategy, we find that the manufacturer always favors the CPS payment scheme if only one of the manufacturer and the retailer adopts the recommender system. When the manufacturer adopts the recommender system, the CPS payment scheme helps mitigate click fraud behaviors and unnecessary costs. Similarly, when the retailer adopts the recommender system, the manufacturer benefits from a "free ride" courtesy of the retailer. Consequently, the manufacturer also finds the CPS payment scheme is advantageous. However, when both the manufacturer and the retailer adopt the recommender system, the manufacturer's preference for the payment scheme depends on the recommendation cost. Specifically, if the recommendation cost is sufficiently small, the manufacturer prefers the CPS payment scheme.

When only the manufacturer adopts the recommender system, all the prices increase with the recommendation strength and cost. The recommender system and the manufacturer always benefit more from a higher recommendation strength but suffer from a higher recommendation cost, even though the recommender system can "shift" the recommendation cost to the manufacturer to some extent. Since the retailer does not adopt the recommender system, its profit remains independent of the recommendation strength and cost. However, in contrast to the scenarios when the retailer does not adopt the recommender system, the manufacturer lowers his wholesale price with an increase in the recommendation strength and cost when only the retailer adopts the recommender system. This can be viewed as a form of "compensation" to prevent the retailer from leaving the market due to both the high wholesale and recommendation costs, ultimately benefiting both the manufacturer and the retailer from the recommendation.

We also derive the win-win situations for the manufacturer and retailer. Regardless of whether the recommender systems uses the CPS or CPC payment scheme, the two firms can achieve a win-

win situation where no firm adopts the recommender system when the recommendation strength is sufficiently low. When the recommendation strength is moderately high and the recommendation cost is sufficiently small, both firms prefer the win-win situation where only the manufacturer adopts the recommender system. We also find that the win-win situation cannot be reached when only the retailer adopts the recommender system, implying that the retailer is unwilling to allow the manufacturer to take a free ride. Regarding the impact of the CPS and CPC payment schemes on win-win situations, we find that the CPC payment scheme requires a lower recommendation cost to enable firms to afford lower transfer costs. In addition, we extend our model, considering simultaneous pricing decisions, outside competition, consumer heterogeneity in channel preferences and co-existence of CPS and CPC payment schemes. The extensions do not affect the qualitative results of the model, but enriches the conclusions of the paper.

Our research provides academics with new research perspectives and research directions in the future. The paper focuses on cost-per-sale (CPS) and cost-per-click (CPC) payment schemes. Other payment models, such as cost-per-thousand impressions (CPM) or value-based pricing, could be considered in future research to provide a more comprehensive understanding of payment scheme impacts. What's more, we assume that consumer preferences are static and known. Incorporating more complex consumer behavior, including the impact of recommendations on brand loyalty, may be more interesting.

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"To Adopt an Online Recommender System? A Manufacturer's Strategic Choice in a Dual-Channel Setting"

Online Appendix A Additional Definitions and Results

A.1 Additional Definitions and Results in Section 3.1

$$
\begin{aligned}
\pi_m^{SN} &= \frac{c^2(1-\theta^2)^2+\alpha^2(41-46\theta-13\theta^2+22\theta^3-3\theta^4)-2c\alpha(5-3\theta-6\theta^2+3\theta^3+\theta^4)}{32(2-\theta^2)(1-\theta^2)} \\
&\quad +\frac{-2(3c(-1+\theta^2)+\alpha(15-17\theta+\theta^2+8\theta^3-4\theta^4))\hat{\alpha}+(9+4\theta^2-4\theta^4)\hat{\alpha}^2}{32(2-\theta^2)(1-\theta^2)}, \\
\pi_r^{SN} &= \frac{4(c(-1+\theta^2)+\alpha(-3+\theta+\theta^2)+(5-2\theta^2)\hat{\alpha})(c(-1+\theta^2)-\alpha(-3+\theta+\theta^2)+(5-2\theta^2)\hat{\alpha})}{64(2-\theta^2)(1-\theta^2)} \\
&\quad +\frac{(c(1-\theta^2)+\alpha(-5+3\theta+\theta^2)+3\hat{\alpha})(3c(-1+\theta^2)-\alpha(-7+\theta+3\theta^2)+(-17+8\theta^2)\hat{\alpha})}{64(2-\theta^2)(1-\theta^2)}, \\
\pi_e^{SN} &= \frac{(c(-1+\theta^2)+\alpha(-3+\theta+\theta^2)+(5-2\theta^2)\hat{\alpha})^2}{16(2-\theta^2)(1-\theta^2)};
\end{aligned}
$$

and

$$
\begin{cases}
\hat{\alpha}_l^{SN} \equiv \max\left\{\frac{c(1-\theta^2)+\alpha(3-\theta-\theta^2)}{5-2\theta^2}, \tilde{\alpha}_1^{SN}, 0\right\}, \\
\hat{\alpha}_h^{SN} \equiv \min\left\{\frac{-c(1-\theta^2)+\alpha(5-\theta-2\theta^2)}{3-\theta^2}, \frac{-c(1-\theta^2)+\alpha(13-7\theta+11\theta^2+4\theta^3+2\theta^4)}{11-10\theta^2+2\theta^4}, \tilde{\alpha}_2^{SN}, 1\right\},
\end{cases}
$$

where

$$
\begin{aligned}
\tilde{\alpha}_1^{SN} &= \frac{-4(1-\theta)\sqrt{2\alpha^2(-8-2\theta+4\theta^2+\theta^3)-3c\alpha(-8-10\theta+2\theta^2+5\theta^3+\theta^4)+c^2(1+\theta)^2(20-13\theta^2+2\theta^4)}}{151-104\theta^2+16\theta^4} \\
&\quad +\frac{c(-13+17\theta^2-4\theta^4)+\alpha(113-47\theta-77\theta^2+20\theta^3+12\theta^4)}{151-104\theta^2+16\theta^4}, \\
\tilde{\alpha}_2^{SN} &= \frac{4(1-\theta)\sqrt{2\alpha^2(-8-2\theta+4\theta^2+\theta^3)-3c\alpha(-8-10\theta+2\theta^2+5\theta^3+\theta^4)+c^2(1+\theta)^2(20-13\theta^2+2\theta^4)}}{151-104\theta^2+16\theta^4} \\
&\quad +\frac{c(-13+17\theta^2-4\theta^4)+\alpha(113-47\theta-77\theta^2+20\theta^3+12\theta^4)}{151-104\theta^2+16\theta^4}.
\end{aligned}
$$

A.2 Additional Definitions and Results in Section 3.2

$$
\begin{aligned}
\pi_m^{SS} &= \frac{c^2(\theta+3)^2-2c(\alpha+\hat{\alpha})(3+\theta)^2+\hat{\alpha}^2(-151-26\theta+\theta^2)}{64(3+\theta)(1+\theta)} \\
&\quad +\frac{2\alpha\hat{\alpha}(73+6\theta+\theta^2)+\alpha^2(41+38\theta+\theta^2)}{64(3+\theta)(1+\theta)}, \\
\pi_r^{SS} &= \frac{-(3+\theta)(\hat{\alpha}(-13+\theta)-c(3+\theta)+\alpha(3+\theta))(\hat{\alpha}(-5+\theta)-c(3+\theta)+\alpha(11+\theta))}{128(3+\theta)^2(1+\theta)} \\
&\quad +\frac{4(\alpha(-5+\theta)+c(3+\theta)+\hat{\alpha}(11+\theta))(c(3+\theta)+\alpha(13+7\theta)-\hat{\alpha}(19+9\theta))}{128(3+\theta)^2(1+\theta)}, \\
\pi_e^{SS} &= \frac{(\alpha(\theta-11)-c(\theta+3)+(\theta+11)\hat{\alpha})^2}{32(3+\theta)(1+\theta)};
\end{aligned}
$$

and

$$\begin{cases} \hat{\alpha}_l^{SS} \equiv \max\left\{\dfrac{c(3+\theta)+\alpha(13+7\theta)}{19+9\theta}, \tilde{\alpha}_1^{SS}, \tilde{\alpha}_3^{SS}, 0\right\}, \\ \hat{\alpha}_h^{SS} \equiv \min\left\{\dfrac{-c(3+\theta)+\alpha(35+9\theta)}{29+7\theta}, \tilde{\alpha}_2^{SS}, \tilde{\alpha}_4^{SS}, 1\right\}. \end{cases}$$

where

$$\begin{aligned}
\tilde{\alpha}_1^{SS} &= \frac{c(-129-109\theta-19\theta^2+\theta^3)+\alpha(713+317\theta-5\theta^2-\theta^3)}{1031+483\theta++21\theta^2+\theta^3} - \frac{8(3+\theta)\sqrt{(3+\theta)(-16\alpha^2(-5+\theta)+c^2(15+2\theta-\theta^2)+2c\alpha(13+18\theta+\theta^2))}}{1031+483\theta++21\theta^2+\theta^3}, \\
\tilde{\alpha}_2^{SS} &= \frac{c(-129-109\theta-19\theta^2+\theta^3)+\alpha(713+317\theta-5\theta^2-\theta^3)}{1031+483\theta++21\theta^2+\theta^3} + \frac{8(3+\theta)\sqrt{(3+\theta)(-16\alpha^2(-5+\theta)+c^2(15+2\theta-\theta^2)+2c\alpha(13+18\theta+\theta^2))}}{1031+483\theta++21\theta^2+\theta^3}, \\
\tilde{\alpha}_3^{SS} &= \frac{-c(3+\theta)^2+\alpha(73+6\theta+\theta^2-\theta^3)}{151+26\theta-\theta^2} - \frac{4\sqrt{2}(3+\theta)\sqrt{(40\alpha^2+c^2(5+\theta)-2c\alpha(7+\theta))}}{151+26\theta-\theta^2}, \\
\tilde{\alpha}_4^{SS} &= \frac{-c(3+\theta)^2+\alpha(73+6\theta+\theta^2-\theta^3)}{151+26\theta-\theta^2} + \frac{4\sqrt{2}(3+\theta)\sqrt{(40\alpha^2+c^2(5+\theta)-2c\alpha(7+\theta))}}{151+26\theta-\theta^2}.
\end{aligned}$$

A.3 Additional Definitions and Results in Section 3.3

$$\begin{aligned}
\pi_m^{CN} &= \frac{k^2\hat{\alpha}^2(\hat{\alpha}^2(9+4\theta^2-4\theta^4)+\alpha^2(41-46\theta-13\theta^2+22\theta^3-3\theta^4)+2\alpha\hat{\alpha}(-15+17\theta-\theta^2-8\theta^3-4\theta^4))}{32k^2\hat{\alpha}^2(2-\theta^2)(1-\theta^2)} \\
&\quad + \frac{c^2(1-\theta^2)^2+2ck\hat{\alpha}(1-\theta^2)(3\hat{\alpha}+\alpha(-5+3\theta+\theta^2))}{32k^2\hat{\alpha}^2(2-\theta^2)(1-\theta^2)}, \\
\pi_r^{CN} &= \frac{k^2\hat{\alpha}^4(-151+104\theta^2-16\theta^4)+k^2\hat{\alpha}^2\alpha^2(-71+50\theta+39\theta^2-18\theta^3-7\theta^4)}{64k^2\hat{\alpha}^2(2-\theta^2)(1-\theta^2)} \\
&\quad + \frac{2k^2\hat{\alpha}^3\alpha(113-47\theta-77\theta^2+20\theta^3+12\theta^4)+c^2(1-\theta^2)^2}{64k^2\hat{\alpha}^2(2-\theta^2)(1-\theta^2)} + \frac{2ck\hat{\alpha}(1-\theta^2)(\alpha(11-5\theta-3\theta^2)-\hat{\alpha}(13-4\theta^2))}{64k^2\hat{\alpha}^2(2-\theta^2)(1-\theta^2)}, \\
\pi_e^{CN} &= \frac{(c(-1+\theta^2)+k\hat{\alpha}(\hat{\alpha}(5-2\theta^2)+\alpha(-3+\theta+\theta^2)))^2}{16k^2\hat{\alpha}^2(2-\theta^2)(1-\theta^2)};
\end{aligned}$$

and

$$\begin{aligned}
c_l^{CN} &\equiv \max\left\{\frac{k\hat{\alpha}(\hat{\alpha}(-5+2\theta^2)+\alpha(3-\theta-\theta^2))}{1-\theta^2}, \tilde{c}_1^{CN}, 0, \frac{k\hat{\alpha}(\hat{\alpha}(-3-2\theta+\theta^2+\theta^3)+\alpha(1+\theta-\theta^3))}{1-\theta^2}\right\}, \\
c_h^{CN} &\equiv \min\left\{\frac{k\hat{\alpha}(\hat{\alpha}(-3+\theta^2)+\alpha(5-\theta-2\theta^2))}{1-\theta^2}, \frac{k\hat{\alpha}(\hat{\alpha}(5-2\theta^2)+\alpha(-3+\theta+\theta^2))}{1-\theta^2}, \frac{k\hat{\alpha}(\hat{\alpha}(-11+10\theta^2-2\theta^4)+\alpha(13-7\theta-11\theta^2+4\theta^3+2\theta^4))}{1-\theta^2}, \tilde{c}_2^{CN}, 1\right\}.
\end{aligned}$$

Moreover,

$$\begin{aligned}
\tilde{c}_1^{CN} &= \frac{-4k\hat{\alpha}\sqrt{\hat{\alpha}^2(20-13\theta^2+2\theta^4)+\alpha^2(12-10\theta-5\theta^2+3\theta^3+\theta^4)-\alpha\hat{\alpha}(32-14\theta-20\theta^2+5\theta^3+3\theta^4)}}{1-\theta^2} + \frac{k\hat{\alpha}(\hat{\alpha}(13-4\theta^2)-\alpha(11-5\theta-3\theta^2))}{1-\theta^2}, \\
\tilde{c}_2^{CN} &= \frac{4k\hat{\alpha}\sqrt{\hat{\alpha}^2(20-13\theta^2+2\theta^4)+\alpha^2(12-10\theta-5\theta^2+3\theta^3+\theta^4)-\alpha\hat{\alpha}(32-14\theta-20\theta^2+5\theta^3+3\theta^4)}}{1-\theta^2} + \frac{k\hat{\alpha}(\hat{\alpha}(13-4\theta^2)-\alpha(11-5\theta-3\theta^2))}{1-\theta^2}.
\end{aligned}$$

A.4 Additional Definitions and Results in Section 3.4

$$\begin{aligned}
\pi_m^{CC} &= \frac{k^2\hat{\alpha}^2\left(\alpha^2\left(\theta^2+38\theta+41\right)+\left(\theta^2-26\theta-151\right)\hat{\alpha}^2+2\alpha\left(\theta^2+6\theta+73\right)\hat{\alpha}\right)}{64k^2\hat{\alpha}^2(\theta+1)(\theta+3)} + \frac{c^2(\theta+3)^2-2c(\theta+3)^2k\hat{\alpha}(\alpha+\hat{\alpha})}{64k^2\hat{\alpha}^2(\theta+1)(\theta+3)}, \\
\pi_r^{CC} &= \frac{(\theta+3)(c(\theta+3)-k\hat{\alpha}(\alpha(\theta+3)+(\theta-13)\hat{\alpha}))(k\hat{\alpha}(\alpha(\theta+11)+(\theta-5)\hat{\alpha})-c(\theta+3))}{128k^2\hat{\alpha}^2(\theta+1)(\theta+3)^2} \\
&\quad + \frac{4(c(\theta+3)+k\hat{\alpha}(\alpha(\theta-5)+(\theta+11)\hat{\alpha}))(c(\theta+3)+k\hat{\alpha}(\alpha(7\theta+13)-(9\theta+19)\hat{\alpha}))}{128k^2\hat{\alpha}^2(\theta+1)(\theta+3)^2}, \\
\pi_e^{CC} &= \frac{(c(\theta+3)-k\hat{\alpha}(\alpha(\theta-5)+(\theta+11)\hat{\alpha}))^2}{32k^2\hat{\alpha}^2(\theta+1)(\theta+3)};
\end{aligned}$$

and

$$
\begin{aligned}
c_l^{CC} &\equiv \max\{\tfrac{k\hat{\alpha}[\alpha(\theta^2+26\theta+37)+(\theta^2-22\theta-43)\hat{\alpha}]}{(\theta-1)(\theta+3)} - \tfrac{8k\hat{\alpha}(\theta^2+2\theta-3)\sqrt{(\alpha-\hat{\alpha})(\alpha(\theta+3)+(\theta-5)\hat{\alpha})}}{(\theta-1)^2(\theta+3)}, 0,\\
&\quad \tfrac{k\hat{\alpha}(\alpha(5-\theta)-\hat{\alpha}(11+\theta))}{3+\theta}, k\hat{\alpha}\left[\alpha + \tfrac{\hat{\alpha}(-13+\theta)}{3+\theta}\right]\},\\
c_h^{CC} &\equiv \min\left\{\tfrac{k\hat{\alpha}(-\alpha(13+7\theta)+\hat{\alpha}(19+9\theta))}{3+\theta}, 1\right\}.
\end{aligned}
$$

Online Appendix B The Retailer and Recommender System's Preferences

In the main text, we have analyzed the manufacturer's strategic decisions concerning the CPS and CPC payment schemes within a dual-channel context. This section examines the preferences of the retailer and recommender system regarding the payment schemes for recommendations. The subsequent proposition initially outlines the retailer's preferences for CPS over CPC payment schemes.

Proposition 9 *When the manufacturer does not adopt the recommender system, the retailer prefers the CPS payment scheme if recommendation cost* $c < 2k\hat{\alpha}\left[\hat{\alpha}(13-4\theta^2) - \alpha(11-5\theta-3\theta^2)\right]/(1+k\hat{\alpha})(1-\theta^2)$*, and CPC payment scheme otherwise. Conversely, when the manufacturer adopts the recommender system, the retailer prefers the CPS payment scheme if recommendation cost* $c < 2k\hat{\alpha}[\hat{\alpha}(22\theta+43-\theta^2) - \alpha(\theta^2+26\theta+37)]/(1+k\hat{\alpha})(1-\theta)(\theta+3)$*, and CPC payment scheme otherwise.*

Proposition 9 demonstrates that regardless of whether the manufacturer implements a recommender system, the retailer prefers the CPS payment scheme when the recommendation cost is relatively low. Conversely, the retailer chooses the CPC payment scheme when the recommendation cost is relatively high. With a low-cost recommender system, the retailer can effectively manage and bear the potential costs associated with product recommendations. This can be attributed to economies of scale resulting from low costs: when recommendation costs are minimal, the additional cost per unit of sale becomes a negligible percentage. The retailer can allocate the recommendation cost proportionally to the actual sales generated through CPS payment scheme. However, when the recommender system incurs higher recommendation costs, the retailer must rigorously manage and optimize the investment in recommendations. At this point, under the CPS payment scheme, the retailer would be responsible for covering the costs associated with all incremental sales. In contrast, the CPC payment scheme allows the retailer to purchase a specific level of exposure to attract potential customers via clicks. Even if the initial conversion rate is low, setting a cost cap enables the retailer to achieve long-term benefits.

Next, the following proposition examines the recommender system's preferences.

Proposition 10 *The recommender systems generate higher profits under the CPS payment scheme compared to the CPC payment scheme.*

The revenue of CPS is positively correlated with sales volume, which is jointly influenced by the strength of recommendations and the manufacturer's pricing strategy. By dynamically adjusting the price of CPS services, the recommender system can effectively balance its profitability with the manufacturer's adoption willingness. For instance, when recommendation strength is high, the recommender system can increase the CPS price while still attracting manufacturers to adopt, as the high conversion rate compensates for the increased cost. Under the CPC payment scheme, the revenue of the recommender system is contingent upon the number of clicks. However, due to the potentially high cost per click, manufacturers might decrease their reliance on the recommender system or even terminate cooperation, leading to unstable revenue for the system. Despite this, some recommender systems adopt the CPC payment scheme in practice. This can be attributed to the fact that the CPS payment scheme aligns better with merchants whose primary objective is sales. In contrast, certain brands prioritize exposure over immediate conversions. For instance, luxury brands frequently opt for CPC schemes to enhance brand awareness rather than focusing solely on short-term sales performance.

Online Appendix C Proofs

Proof of equilibrium results in Scenario SN. According to inverse induction, due to $\partial^2\pi_r/\partial p_r^2 = -2-2/\left(1-\theta^2\right) < 0$, we have maximum value $p_r = (-\alpha\theta+\alpha-\theta^2 p_e + p_e + \theta p_m - \theta^2\hat{\alpha} + \hat{\alpha} - \theta^2 w + 2w)/(4-2\theta^2)$ by letting $\partial\pi_r/\partial p_r = 0$. Taking the p_r into π_m, we can find the Hessian matrix of π_m with respect to p_m and w as

$$H = \begin{bmatrix} \left(2-\theta^2\right)/\left(-1+\theta^2\right) & \theta/\left(1-\theta^2\right) \\ \left(2-\theta^2\right)/\left(-1+\theta^2\right) & \left(-4+3\theta^2\right)/\left(2-3\theta^2+\theta^4\right) \end{bmatrix}.$$

It is easy to obtain that this Hessian matrix is negative definite. Solving the system of equations

$$\begin{cases} \partial\pi_m/\partial p_m = 0 \\ \partial\pi_m/\partial w = 0 \end{cases}$$

and taking the result into π_e, we have $\partial^2\pi_e/\partial p_e^2 = -\left(1-\theta^2\right)/\left(4-2\theta^2\right) < 0$, which means that π_e has the maximum value with respect to p_e. Letting $\partial\pi_e/\partial p_e = 0$, we can get the equilibrium results. The proof procedure for the other models is similar and we only list the second order partial derivatives and the Hessian matrix. ■

Proof of equilibrium results in Scenario SS. We have $\partial^2\pi_r/\partial p_r^2 = -4/\left(1-\theta^2\right) < 0$. Hessian matrix of π_m with respect to p_m and w

$$H = \begin{bmatrix} 2/\left(-1+\theta^2\right) & 2\theta/\left(1-\theta^2\right) \\ 2\theta/\left(1-\theta^2\right) & \left(-4+2\theta^2\right)/\left(1-\theta^2\right) \end{bmatrix}$$

is negative definite. Furthermore, we have $\partial^2\pi_e/\partial p_e^2 = -\left(3+\theta\right)/\left(4+4\theta\right) < 0$. Therefore, the existence of equilibrium solutions is observed. ■

Proof of equilibrium results in Scenario CN. We obtain that $\partial^2\pi_r/\partial p_r^2 = -2-2/\left(1-\theta^2\right) < 0$.

Hessian matrix of π_m with respect to p_m and w

$$H = \begin{bmatrix} \left(2-\theta^2\right)/\left(-1+\theta^2\right) & \theta/\left(1-\theta^2\right) \\ \left(2-\theta^2\right)/\left(-1+\theta^2\right) & \left(-4+3\theta^2\right)/\left(2-3\theta^2+\theta^4\right) \end{bmatrix}$$

is negative definite. Moreover, we also have $\partial^2\pi_e/\partial p_e^2 = -\left(1-\theta^2\right)/\left[2k^2\hat{\alpha}^2(2-\theta^2)\right] < 0$. Therefore, the existence of equilibrium solutions is observed. ■

Proof of equilibrium results in Scenario CC. We have $\partial^2\pi_r/\partial p_r^2 = -4/\left(1-\theta^2\right) < 0$. Hessian matrix of π_m with respect to p_m and w

$$H = \begin{bmatrix} 2/\left(-1+\theta^2\right) & 2\theta/\left(1-\theta^2\right) \\ 2\theta/\left(1-\theta^2\right) & \left(-4+2\theta^2\right)/\left(1-\theta^2\right) \end{bmatrix}$$

is also negative definite. Furthermore, we obtain that $\partial^2\pi_e/\partial p_e^2 = -(3+\theta)/\left[4k^2\hat{\alpha}^2(1+\theta)\right] < 0$. Therefore, the existence of equilibrium solutions is observed. ■

Proof of equilibrium results in Scenario NN. We obtain that $\partial^2\pi_r/\partial p_r^2 = -2/\left(1-\theta^2\right) < 0$. Obviously, we find that Hessian matrix of π_m with respect to p_m and w

$$H = \begin{bmatrix} 1/\left(-1+\theta^2\right) & \theta/\left(1-\theta^2\right) \\ \theta/\left(1-\theta^2\right) & \left(-2+\theta^2\right)/\left(1-\theta^2\right) \end{bmatrix}$$

is negative definite. Thus, there is an equilibrium solution. ■

Proof of equilibrium results in Scenario NS. We have $\partial^2\pi_r/\partial p_r^2 = -2/\left(1-\theta^2\right) < 0$. Hessian matrix of π_m with respect to p_m and w

$$H = \begin{bmatrix} 1/\left(-1+\theta^2\right) & \theta/\left(1-\theta^2\right) \\ \theta/\left(1-\theta^2\right) & \left(-4+3\theta^2\right)/\left(1-\theta^2\right) \end{bmatrix}$$

is also negative definite. Furthermore, we also have $\partial^2\pi_e/\partial p_e^2 = -1/2 < 0$. Therefore, the existence of equilibrium solutions is observed. ■

Proof of equilibrium results in Scenario NC. We derive that $\partial^2\pi_r/\partial p_r^2 = -2/\left(1-\theta^2\right) < 0$. Hessian matrix of π_m with respect to p_m and w

$$H = \begin{bmatrix} 1/\left(-1+\theta^2\right) & \theta/\left(1-\theta^2\right) \\ \theta/\left(1-\theta^2\right) & \left(-4+3\theta^2\right)/\left(1-\theta^2\right) \end{bmatrix}$$

is negative definite. And then we have $\partial^2\pi_e/\partial p_e^2 = -1/2k^2\hat{\alpha}^2 < 0$. Therefore, the existence of equilibrium solutions can be observed. ■

Proof of Proposition 1. First, we compute

$$\frac{\partial p_m^{SN}}{\partial \hat{\alpha}} = \frac{\theta}{4} > 0, \frac{\partial w^{SN}}{\partial \hat{\alpha}} = \frac{3-\theta^2}{-4(2-\theta^2)} < 0, \ \frac{\partial p_e^{SN}}{\partial \hat{\alpha}} = \frac{5-2\theta^2}{2(1-\theta^2)} > 0, \text{ and } \frac{\partial p_r^{SN}}{\partial \hat{\alpha}} = \frac{11-6\theta^2}{8(2-\theta^2)} > 0.$$

Next, we calculate

$$\frac{\partial p_m^{SN}}{\partial c} = 0, \ \frac{\partial w^{SN}}{\partial c} = \frac{1-\theta^2}{-4(2-\theta^2)} < 0, \ \frac{\partial p_e^{SN}}{\partial c} = \frac{1}{2} > 0, \text{ and} \frac{\partial p_r^{SN}}{\partial c} = \frac{1-\theta^2}{8(2-\theta^2)} > 0.$$

■

Proof of Table 2. We calculate

$$\frac{\partial \pi_m^{SN}}{\partial \hat{\alpha}} = \frac{3c(1-\theta^2) + \alpha(-15+17\theta-\theta^2-8\theta^3+4\theta^4) + (9+4\theta^2-4\theta^4)\hat{\alpha}}{16(2-3\theta^2+\theta^4)},$$

and obtain a threshold $\hat{\alpha}_1^{SN} = [-3c(1-\theta^2)+\alpha(15-17\theta+\theta^2+8\theta^3-4\theta^4)]/(9+4\theta^2-4\theta^4)$. If $\hat{\alpha} > \hat{\alpha}_3^{SN}$, $\partial \pi_m^{SN}/\partial \hat{\alpha} > 0$; otherwise, $\partial \pi_m^{SN}/\partial \hat{\alpha} < 0$. However, we cannot determine whether $\hat{\alpha}_1^{SN}$ is between 0 and 1. Thus, when $\hat{\alpha}_1^{SN} < 0$, $\partial \pi_m^{SN}/\partial \hat{\alpha} > 0$; when $\hat{\alpha}_1^{SN} > 1$, $\partial \pi_m^{SN}/\partial \hat{\alpha} < 0$; when $0 < \hat{\alpha}_1^{SN} < 1$, $\partial \pi_m^{SN}/\partial \hat{\alpha} > 0$ if $\hat{\alpha} > \hat{\alpha}_1^{SN}$; $\partial \pi_m^{SN}/\partial \hat{\alpha} < 0$ if $\hat{\alpha} < \hat{\alpha}_1^{SN}$.

We also compute

$$\frac{\partial \pi_r^{SN}}{\partial \hat{\alpha}} = \frac{c(-13+17\theta^2-4\theta^4) + \alpha(113-47\theta-77\theta^2+20\theta^3+12\theta^4) + (-151+104\theta^2-16\theta^4)\hat{\alpha}}{32(2-3\theta^2+\theta^4)},$$

and find a threshold $\hat{\alpha}_2^{SN} = [c(-13+17\theta^2-4\theta^4)+\alpha(113-47\theta-77\theta^2+20\theta^3+12\theta^4)]/(151-104\theta^2+16\theta^4)$. If $\hat{\alpha} > \hat{\alpha}_2^{SN}$, $\partial \pi_r^{SN}/\partial \hat{\alpha} < 0$; otherwise, $\partial \pi_r^{SN}/\partial \hat{\alpha} > 0$.

Moreover,

$$\frac{\partial \pi_e^{SN}}{\partial \hat{\alpha}} = \frac{(5-2\theta^2)(c(-1+\theta^2) + \alpha(-3+\theta+\theta^2) + (5-2\theta^2)\hat{\alpha})}{8(2-3\theta^2+\theta^4)},$$

which is positive when $\hat{\alpha} \in \left(\hat{\alpha}_l^{SN}, \hat{\alpha}_h^{SN}\right)$. We also find

$$\frac{\partial \pi_m^{SN}}{\partial c} = \frac{c(1-\theta^2) + \alpha(\theta^2+3\theta-5) + 3\hat{\alpha}}{16(2-\theta^2)},$$

and obtain a threshold $\hat{\alpha}_3^{SN} = \frac{1}{3}(c(-1+\theta^2)+\alpha(5-3\theta-\theta^2))$. If $\hat{\alpha} > \hat{\alpha}_3^{SN}$, $\partial \pi_m^{SN}/\partial c > 0$; otherwise, $\partial \pi_m^{SN}/\partial c < 0$.

Then, we differentiate π_r^{SN} once w.r.t. c as

$$\frac{\partial \pi_r^{SN}}{\partial c} = \frac{c(1-\theta^2) + \alpha(11-5\theta-3\theta^2) - (13-4\theta^2)\hat{\alpha}}{32(2-\theta^2)},$$

and find that, if $\hat{\alpha} > \hat{\alpha}_4^{SN} \equiv [c(1-\theta^2)+\alpha(11-5\theta-3\theta^2)]/(13-4\theta^2)$, $\partial \pi_r^{SN}/\partial c < 0$; otherwise,

$\partial\pi_r^{SN}/\partial c > 0$. In addition,

$$\frac{\partial \pi_e^{SN}}{\partial c} = \frac{c(1-\theta^2) + \alpha(3-\theta-\theta^2) - (5-2\theta^2)\hat{\alpha}}{8(2-\theta^2)},$$

which is negative when $\hat{\alpha} \in \left(\hat{\alpha}_l^{SN}, \hat{\alpha}_h^{SN}\right)$. ■

Proof of Proposition 2 . We can easily get the following:

$$\begin{aligned}
\frac{\partial D_m^{SS}}{\partial \hat{\alpha}} &= \frac{-34-11\theta+\theta^2}{16(1+\theta)(3+\theta)} < 0, \\
\frac{\partial D_r^{SS}}{\partial \hat{\alpha}} &= \frac{-29-7\theta}{16(1+\theta)(3+\theta)} < 0, \\
\frac{\partial \hat{D}_m^{SS}}{\partial \hat{\alpha}} &= \frac{14+5\theta+\theta^2}{16(1+\theta)(3+\theta)} > 0, \\
\frac{\partial \hat{D}_r^{SS}}{\partial \hat{\alpha}} &= \frac{19+9\theta}{16(1+\theta)(3+\theta)} > 0.
\end{aligned}$$

■

Proof of Table 3. Using the profit functions for scenario "SS," we have

$$\frac{\partial \pi_m^{SS}}{\partial \hat{\alpha}} = \frac{-c(3+\theta)^2 + (-151-26\theta+\theta^2)\hat{\alpha} + \alpha(73+6\theta+\theta^2)}{32(1+\theta)(3+\theta)}.$$

Then, if $\hat{\alpha} < \hat{\alpha}_1^{SS} \equiv [-c(3+\theta)^2 + \alpha(73+6\theta+\theta^2)]/(151+26\theta-\theta^2)$, $\partial\pi_m^{SS}/\partial\hat{\alpha} > 0$; otherwise $\partial\pi_m^{SS}/\partial\hat{\alpha} < 0$. Moreover, we calculate

$$\begin{aligned}
\frac{\partial \pi_r^{SS}}{\partial \hat{\alpha}} &= -\frac{1}{64(1+\theta)(3+\theta)^2}[c(129+109\theta+19\theta^2-\theta^3) \\
&\quad +\alpha(-173-317\theta+5\theta^2+\theta^3) + \hat{\alpha}(1031+483\theta+21\theta^2+\theta^3)],
\end{aligned}$$

and find that, if $\hat{\alpha} < \hat{\alpha}_2^{SS} \equiv [-c(129+109\theta+19\theta^2-\theta^3) - \alpha(-173-317\theta+5\theta^2+\theta^3)]/(1031+483\theta+21\theta^2+\theta^3)$, $\partial\pi_r^{SS}/\partial\hat{\alpha} > 0$; otherwise, $\partial\pi_r^{SS}/\partial\hat{\alpha} < 0$.

As

$$\frac{\partial \pi_e^{SS}}{\partial \hat{\alpha}} = \frac{(11+\theta)(\alpha(-5+\theta) - c(3+\theta) + \hat{\alpha}(11+\theta))}{16(1+\theta)(3+\theta)},$$

which is positive when $\hat{\alpha} \in \left(\hat{\alpha}_l^{SS}, \hat{\alpha}_h^{SS}\right)$. In addition,

$$\frac{\partial \pi_m^{SS}}{\partial c} = \frac{(c-\hat{\alpha}-\alpha)(3+\theta)}{32(1+\theta)},$$

and find that, if $\hat{\alpha} < \hat{\alpha}_3^{SS} \equiv c - \alpha$, $\partial\pi_m^{SS}/\partial c > 0$; otherwise $\partial\pi_m^{SS}/\partial c < 0$.

We also have

$$\frac{\partial \pi_r^{SS}}{\partial c} = \frac{c(3-2\theta-\theta^2) + \alpha(37+26\theta+\theta^2) + \hat{\alpha}(-43-22\theta+\theta^2)}{64(1+\theta)(3+\theta)},$$

and find that, if $\hat{\alpha} < \hat{\alpha}_4^{SS} \equiv [c(3-2\theta-\theta^2)+\alpha(37+26\theta+\theta^2)]/(43+22\theta-\theta^2)$, $\partial\pi_r^{SS}/\partial c > 0$; otherwise $\partial\pi_r^{SS}/\partial c < 0$. Moreover,

$$\frac{\partial\pi_e^{SS}}{\partial c} = \frac{\alpha(5-\theta)+c(3+\theta)-\hat{\alpha}(11+\theta)}{16(1+\theta)},$$

which is negative when $\hat{\alpha} \in (\hat{\alpha}_l^{SS}, \hat{\alpha}_h^{SS})$. ■

Proof of Table 4. We first compute $\partial p_m^{CN}/\partial\hat{\alpha} = \theta/4 > 0$ and then calculate

$$\frac{\partial w^{CN}}{\partial\hat{\alpha}} = \frac{c(1-\theta^2)-k\hat{\alpha}^2(3-\theta^2)}{4k\hat{\alpha}^2(2-\theta^2)}.$$

If $c > c_1^{CN} = k\hat{\alpha}^2(3-\theta^2)/(1-\theta^2)$, then $\partial w^{CN}/\partial\hat{\alpha} > 0$; if $c < c_1^{CN}$, then $\partial w^{CN}/\partial\hat{\alpha} < 0$. We also find

$$\frac{\partial p_e^{CN}}{\partial\hat{\alpha}} = \frac{k[\alpha(-3+\theta+\theta^2)+(10-4\theta^2)\hat{\alpha}]}{2(1-\theta^2)}.$$

If $\hat{\alpha} > \hat{\alpha}_1^{CN} = \alpha(3-\theta-\theta^2)/(10-4\theta^2)$, then $\partial p_e^{CN}/\partial\hat{\alpha} > 0$; if $\hat{\alpha} < \hat{\alpha}_1^{CN}$, $\partial p_e^{CN}/\partial\hat{\alpha} < 0$. Since

$$\frac{\partial p_r^{CN}}{\partial\hat{\alpha}} = \frac{c(-1+\theta^2)+k\hat{\alpha}^2(11-6\theta^2)}{8k\hat{\alpha}^2(2-\theta^2)},$$

we find that if $c < c_2^{CN} = k\hat{\alpha}^2(11-6\theta^2)/(1-\theta^2)$, $\partial p_r^{CN}/\partial\hat{\alpha} > 0$ if $c > c_2^{CN}$, $\partial p_r^{CN}/\partial\hat{\alpha} < 0$.

Next, we obtain

$$\frac{\partial p_m^{CN}}{\partial c} = 0 \text{ and } \frac{\partial w^{CN}}{\partial c} = \frac{1-\theta^2}{-4k\hat{\alpha}(2-\theta^2)} < 0;$$

and

$$\frac{\partial p_e^{CN}}{\partial c} = \frac{1}{2} > 0 \text{ and } \frac{\partial p_r^{CN}}{\partial c} = \frac{1-\theta^2}{8k\hat{\alpha}(2-\theta^2)} > 0.$$

■

Proof of Table 5. We have

$$\begin{aligned}\frac{\partial\pi_m^{CN}}{\partial\hat{\alpha}} &= \frac{-c^2(1-\theta^2)^2+ck\alpha\hat{\alpha}(5-3\theta-6\theta^2+3\theta^3+\theta^4)}{16k^2\hat{\alpha}^3(2-3\theta^2+\theta^4)} \\ &\quad + \frac{k^2\alpha\hat{\alpha}^3(-15+17\theta-\theta^2-8\theta^3+4\theta^4)+k^2\hat{\alpha}^4(9+4\theta^2-4\theta^4)}{16k^2\hat{\alpha}^3(2-3\theta^2+\theta^4)}.\end{aligned}$$

If $c_3^{CN} < c < c_4^{CN}$, where

$$\begin{aligned} c_3^{CN} &\equiv \frac{-k\hat{\alpha}\sqrt{\hat{\alpha}^2(5-3\theta-\theta^2)^2-4\alpha\hat{\alpha}(15-17\theta+\theta^2+8\theta^3-4\theta^4)+4\hat{\alpha}^2(9+4\theta^2-4\theta^4)}}{2(1-\theta^2)} + \frac{k\alpha\hat{\alpha}(5-3\theta-\theta^2)}{2(1-\theta^2)}, \\ c_4^{CN} &\equiv \frac{k\hat{\alpha}\sqrt{\hat{\alpha}^2(5-3\theta-\theta^2)^2-4\alpha\hat{\alpha}(15-17\theta+\theta^2+8\theta^3-4\theta^4)+4\hat{\alpha}^2(9+4\theta^2-4\theta^4)}}{2(1-\theta^2)} + \frac{k\alpha\hat{\alpha}(5-3\theta-\theta^2)}{2(1-\theta^2)}, \end{aligned}$$

then $\partial\pi_m^{CN}/\partial\hat{\alpha} > 0$; otherwise, $\partial\pi_m^{CN}/\partial\hat{\alpha} < 0$.

As

$$\frac{\partial \pi_m^{CN}}{\partial c} = \frac{c(1-\theta^2) - k\alpha\hat{\alpha}(5-3\theta-\theta^2) + 3k\hat{\alpha}^2}{16k^2\hat{\alpha}^2(2-\theta^2)},$$

we find that if $c > c_8^{CN} \equiv k\hat{\alpha}(-3\hat{\alpha} + \alpha(5-3\theta-\theta^2))/(1-\theta^2)$, then $\partial \pi_m^{CN}/\partial c > 0$; otherwise, then $\partial \pi_m^{CN}/\partial c < 0$. In addition,

$$\begin{aligned}\frac{\partial \pi_r^{CN}}{\partial \hat{\alpha}} &= \frac{-c^2(1-\theta^2)^2 + ck\alpha\hat{\alpha}(-11+5\theta+14\theta^2-5\theta^3-3\theta^4)}{32k^2\hat{\alpha}^3(2-3\theta^2+\theta^4)} \\ &\quad + \frac{k^2\alpha\hat{\alpha}^3(113-47\theta-77\theta^2-20\theta^3+12\theta^4) + k^2\hat{\alpha}^4(-151+104\theta^2-16\theta^4)}{32k^2\hat{\alpha}^3(2-3\theta^2+\theta^4)}.\end{aligned}$$

If $c_5^{CN} < c < c_6^{CN}$, where

$$\begin{aligned} c_5^{CN} &\equiv \frac{-k\hat{\alpha}\sqrt{\hat{\alpha}^2(-11+5\theta+3\theta^2)^2 + 4\alpha\hat{\alpha}(113-47\theta-77\theta^2-20\theta^3+12\theta^4) - \hat{\alpha}^2(604-416\theta^2+64\theta^4)}}{2(1-\theta^2)} + \frac{k\alpha\hat{\alpha}(-11+5\theta+3\theta^2)}{2(1-\theta^2)}, \\ c_6^{CN} &\equiv \frac{k\hat{\alpha}\sqrt{\hat{\alpha}^2(-11+5\theta+3\theta^2)^2 + 4\alpha\hat{\alpha}(113-47\theta-77\theta^2-20\theta^3+12\theta^4) - \hat{\alpha}^2(604-416\theta^2+64\theta^4)}}{2(1-\theta^2)} + \frac{k\alpha\hat{\alpha}(-11+5\theta+3\theta^2)}{2(1-\theta^2)}. \end{aligned}$$

$\partial \pi_r^{CN}/\partial \hat{\alpha} > 0$; otherwise $\partial \pi_r^{CN}/\partial \hat{\alpha} < 0$. Since

$$\frac{\partial \pi_r^{CN}}{\partial c} = \frac{c(1-\theta^2) + k\alpha\hat{\alpha}(11-5\theta-3\theta^2) - k\hat{\alpha}^2(13-4\theta^2)}{32k^2\hat{\alpha}^2(2-\theta^2)},$$

we find that if $c > c_9^{CN} \equiv k\hat{\alpha}(\hat{\alpha}(13-4\theta^2) - \alpha(11-5\theta-3\theta^2))/(1-\theta^2)$, $\partial \pi_r^{CN}/\partial c > 0$; otherwise $\partial \pi_r^{CN}/\partial c < 0$.

As

$$\frac{\partial \pi_e^{CN}}{\partial \hat{\alpha}} = \frac{[c(1-\theta^2) + k\hat{\alpha}^2(5-2\theta^2)][c(-1+\theta^2) + k\hat{\alpha}^2(5-2\theta^2) - k\alpha\hat{\alpha}(3-\theta-\theta^2)]}{8k^2\hat{\alpha}^3(2-3\theta^2+\theta^4)},$$

we find that when $\hat{\alpha} < \hat{\alpha}_2^{CN} \equiv \alpha(3-\theta-\theta^2)/(10-4\theta^2)$, $\partial \pi_e^{CN}/\partial \hat{\alpha} < 0$. When $\hat{\alpha} > \hat{\alpha}_2^{CN}$, if $c < c_7^{CN} \equiv k\hat{\alpha}(\hat{\alpha}(5-2\theta^2) - \alpha(3-\theta-\theta^2))/(1-\theta^2)$, $\partial \pi_e^{CN}/\partial \hat{\alpha} > 0$, otherwise $\partial \pi_e^{CN}/\partial \hat{\alpha} < 0$. We also have

$$\frac{\partial \pi_e^{CN}}{\partial c} = \frac{c(1-\theta^2) + k\alpha\hat{\alpha}(3-\theta-\theta^2) - k\hat{\alpha}^2(5-2\theta^2)}{8k^2\hat{\alpha}^2(2-\theta^2)},$$

and find that if $c > c_{10}^{CN} \equiv k\hat{\alpha}(\hat{\alpha}(5-2\theta^2) - \alpha(3-\theta-\theta^2))/(1-\theta^2)$, $\partial \pi_e^{CN}/\partial c > 0$, otherwise $\partial \pi_e^{CN}/\partial c < 0$. ∎

Proof of Table 6. Using the prices in equilibrium for scenario "CC," we have

$$\frac{\partial p_m^{CC}}{\partial \hat{\alpha}} = \frac{1}{8}\left(-\frac{c}{k\hat{\alpha}^2} + \frac{17+3\theta}{3+\theta}\right).$$

Therefore, if $c < c_1^{CC} \equiv \left[(17+3\theta)\,k\hat{\alpha}^2\right]/(3+\theta)$, $\partial p_m^{CC}/\partial \hat{\alpha} > 0$; otherwise $\partial p_m^{CC}/\partial \hat{\alpha} < 0$. In addition, we have

$$\frac{\partial w^{CC}}{\partial \hat{\alpha}} = \frac{1}{8}\left(\frac{c}{k\hat{\alpha}^2} + \frac{-5+\theta}{3+\theta}\right),$$

and find that, if $c > c_2^{CC} \equiv k\hat{\alpha}^2(5-\theta)/(3+\theta)$, $\partial w^{CC}/\partial\hat{\alpha} > 0$; otherwise $\partial w^{CC}/\partial\hat{\alpha} < 0$.

Since

$$\frac{\partial p_r^{CC}}{\partial\hat{\alpha}} = \frac{k\hat{\alpha}^2(29+12\theta-\theta^2) - c(3+4\theta+\theta^2)}{16k\hat{\alpha}^2(3+\theta)},$$

we find that if $c < c_3^{CC} \equiv k\hat{\alpha}^2(29+12\theta-\theta^2)/(3+4\theta+\theta^2)$, $\partial p_r^{CC}/\partial\hat{\alpha} > 0$; otherwise $\partial p_r^{CC}/\partial\hat{\alpha} < 0$.

We also compute

$$\frac{\partial p_e^{CC}}{\partial\hat{\alpha}} = \frac{k(2\hat{\alpha}(11+\theta)-\alpha(5-\theta))}{2(3+\theta)},$$

which indicates that, if $\hat{\alpha} > \hat{\alpha}^{CC} \equiv \alpha(5-\theta)/(22+2\theta)$, $\partial p_e^{CC}/\partial\hat{\alpha} > 0$; otherwise $\partial p_e^{CC}/\partial\hat{\alpha} < 0$. In addition, $\partial p_m^{CC}/\partial c = 1/(8k\hat{\alpha}) > 0$, $\partial w^{CC}/\partial c = -1/(8k\hat{\alpha}) < 0$, $\partial p_r^{CC}/\partial c = (3+4\theta+\theta^2)/[16k\hat{\alpha}(3+\theta)] > 0$ and $\partial p_e^{CC}/\partial c = 1/2 > 0$. ■

Proof of Table 7. We have

$$\frac{\partial\pi_m^{CC}}{\partial\hat{\alpha}} = \frac{-c^2(3+\theta)^2 + ck\alpha\hat{\alpha}(3+\theta)^2 + k^2\hat{\alpha}^3[\alpha(73+6\theta+\theta^2)+\hat{\alpha}(-151-26\theta+\theta^2)]}{32k^2\hat{\alpha}^3(1+\theta)(3+\theta)},$$

and obtain that $\partial\pi_m^{CC}/\partial\hat{\alpha} > 0$ if $c_4^{CC} < c < c_5^{CC}$ where

$$\begin{aligned} c_4^{CC} &\equiv \frac{k\alpha\hat{\alpha} - k\hat{\alpha}\sqrt{\alpha^2(3+\theta)^2 + 4\hat{\alpha}^2(-151-26\theta+\theta^2) + 4\alpha\hat{\alpha}(73+6\theta+\theta^2)}}{2(3+\theta)}, \\ c_5^{CC} &\equiv \frac{k\alpha\hat{\alpha} + k\hat{\alpha}\sqrt{\alpha^2(3+\theta)^2 + 4\hat{\alpha}^2(-151-26\theta+\theta^2) + 4\alpha\hat{\alpha}(73+6\theta+\theta^2)}}{2(3+\theta)}, \end{aligned}$$

and $\hat{\alpha} < \alpha/2$, or $c_4^{CC} < c < c_5^{CC}$ and $\alpha/2 < \hat{\alpha} < (73+4\sqrt{418})\alpha/302$ and $0 < \theta < (52\hat{\alpha}^2 - 12\alpha\hat{\alpha} - 3\alpha^2 - 16\sqrt{20\hat{\alpha}^3 - 3\alpha^2\hat{\alpha} - \alpha^3})/(2\hat{\alpha}+\alpha)^2$; $\partial\pi_m^{CC}/\partial\hat{\alpha} < 0$ otherwise. For simplicity, we define $\Omega_1 = (0,\alpha/2)\cup((\alpha/2,(73+4\sqrt{418})\alpha/302)\cap(0,(52\hat{\alpha}^2-12\alpha\hat{\alpha}-3\alpha^2-16\sqrt{20\hat{\alpha}^3-3\alpha^2\hat{\alpha}-\alpha^3})/(2\hat{\alpha}+\alpha)^2))$.

As

$$\begin{aligned} \frac{\partial\pi_r^{CC}}{\partial\hat{\alpha}} &= -\frac{c^2(1-\theta)(3+\theta)^2 + ck\alpha\hat{\alpha}(111+115\theta+29\theta^2+\theta^3)^2}{64k^2\hat{\alpha}^3(1+\theta)(3+\theta)^2} \\ &\quad -\frac{k^2\hat{\alpha}^3[\alpha(-713-317\theta+5\theta^2+\theta^3)+\hat{\alpha}(1031+483\theta+21\theta^2+\theta^3)]}{64k^2\hat{\alpha}^3(1+\theta)(3+\theta)^2}, \end{aligned}$$

we have: $\partial\pi_r^{CC}/\partial\hat{\alpha} > 0$ if $c_6^{CC} < c < c_7^{CC}$ where

$$\begin{aligned} c_6^{CC} &\equiv \frac{-k\hat{\alpha}\sqrt{\alpha^2(37+26\theta+\theta^2)^2+4\hat{\alpha}^2(-1031+548\theta+462\theta^2+20\theta^3+\theta^4)+4\alpha\hat{\alpha}(713-396\theta-322\theta^2+4\theta^3+\theta^4)}}{2(1-\theta)(3+\theta)} \\ &\quad + \frac{-k\alpha\hat{\alpha}(37+26\theta+\theta^2)}{2(1-\theta)(3+\theta)}, \\ c_7^{CC} &\equiv \frac{k\hat{\alpha}\sqrt{\alpha^2(37+26\theta+\theta^2)^2+4\hat{\alpha}^2(-1031+548\theta+462\theta^2+20\theta^3+\theta^4)+4\alpha\hat{\alpha}(713-396\theta-322\theta^2+4\theta^3+\theta^4)}}{2(1-\theta)(3+\theta)} \\ &\quad + \frac{-k\alpha\hat{\alpha}(37+26\theta+\theta^2)}{2(1-\theta)(3+\theta)}; \end{aligned}$$

$\partial\pi_r^{CC}/\partial\hat{\alpha} < 0$ otherwise.

We compute

$$\frac{\partial \pi_e^{CC}}{\partial \hat{\alpha}} = \frac{(c(3+\theta)+k\hat{\alpha}^2(11+\theta))(c(3+\theta)-k\hat{\alpha}(\alpha(-5+\theta)+\hat{\alpha}(11+\theta)))}{16k^2\hat{\alpha}^3(1+\theta)(3+\theta)} > 0.$$

Computing

$$\frac{\partial \pi_m^{CC}}{\partial c} = \frac{(3+\theta)[c-k\hat{\alpha}(\hat{\alpha}+\alpha)]}{32k^2\hat{\alpha}^2(1+\theta)},$$

we find that if $c > c_8^{CC} \equiv k\hat{\alpha}(\hat{\alpha}+\alpha)$, $\partial \pi_m^{CC}/\partial c > 0$; otherwise $\partial \pi_m^{CC}/\partial c < 0$. Moreover,

$$\frac{\partial \pi_r^{CC}}{\partial c} = \frac{c(3-2\theta+\theta^2)+k\hat{\alpha}[\hat{\alpha}(\theta^2-22\theta-43)+\alpha(\theta^2+26\theta+37)]}{64k^2\hat{\alpha}^2(1+\theta)(3+\theta)},$$

and, if $c > c_9^{CC} \equiv k\hat{\alpha}[\hat{\alpha}(43+22\theta-\theta^2)-\alpha(4\theta^2+26\theta+37)]/(3-2\theta+\theta^2)$, $\partial \pi_r^{CC}/\partial c > 0$; otherwise $\partial \pi_r^{CC}/\partial c < 0$. In addition

$$\frac{\partial \pi_e^{CC}}{\partial c} = \frac{c(3+\theta)-k\hat{\alpha}(\alpha(11+\theta)+\hat{\alpha}(-5+\theta))}{16k^2\hat{\alpha}^2(1+\theta)} < 0.$$

■

Proof of Theorem 1. Letting $\Delta_m^{SN} = \pi_m^{SN} - \pi_m^{SS}$ and solving $\Delta_m^{SN} = 0$, we find

$$\begin{aligned}
\tilde{\alpha}_{m1} &= \frac{\alpha(236-206\theta-111\theta^2+115\theta^3-3\theta^4-7\theta^5)-\sqrt{2}\sqrt{H}}{356-232\theta-181\theta^2+135\theta^3+3\theta^4-9\theta^5} \\
&\quad + \frac{-c(36-37\theta^2-5\theta^3+5\theta^4+\theta^5)}{356-232\theta-181\theta^2+135\theta^3+3\theta^4-9\theta^5}, \\
\tilde{\alpha}_{m2} &= \frac{\alpha(236-206\theta-111\theta^2+115\theta^3-3\theta^4-7\theta^5)+\sqrt{2}\sqrt{H}}{356-232\theta-181\theta^2+135\theta^3+3\theta^4-9\theta^5} \\
&\quad + \frac{-c(36-37\theta^2-5\theta^3+5\theta^4+\theta^5)}{356-232\theta-181\theta^2+135\theta^3+3\theta^4-9\theta^5},
\end{aligned}$$

where

$$\begin{aligned}
H \equiv\; & \alpha^2(1-\theta)^2(448-96\theta-434\theta^2+84\theta^3+119\theta^4-18\theta^5-7\theta^6) \\
& -c^2(928-320\theta-816\theta^2+368\theta^3+46\theta^4-124\theta^5+65\theta^6+2\theta^7-5\theta^8) \\
& +2c\alpha(704-184\theta-360\theta^2+54\theta^3-2\theta^4-91\theta^5-3\theta^6+9\theta^7+\theta^8).
\end{aligned}$$

Because Δ_m^{SN} is a quadratic function of $\hat{\alpha}$ and $\partial^2\Delta_m^{SN}/\partial\hat{\alpha}^2 = (7\theta+53)/8(\theta^2+5\theta+4)+(-2\theta^4+3\theta^2+1)/(9\theta^4-26\theta^2+16) > 0$, Δ_m^{SN} first decreases and then increases in $\hat{\alpha}$. Because $\tilde{\alpha}_{m2} < \hat{\alpha}_l^{SN}$, $\Delta_m^{SN} > 0$.

In addition, letting $\Delta_m^{CN} \equiv \pi_m^{CN} - \pi_m^{CC}$ and solving $\Delta_m^{CN} = 0$, we obtain $\tilde{c}_{m1}$ and $\tilde{c}_{m2}$. Because Δ_m^{CN} is a quadratic function of c and $\partial^2\Delta_m^{CN}/\partial\hat{\alpha}^2 = -(4-\theta^2+\theta^3)/[32k^2\hat{\alpha}^2(2-\theta^2)(1+\theta)] < 0$, Δ_m^{CN} first increases and then decreases in c. Therefore, if $c \in (\tilde{c}_{m1}, \tilde{c}_{m2}) \cap F_c$, $\Delta_m^{CN} > 0$; otherwise $\Delta_m^{CN} < 0$. When $c = 0$, $\Delta_m^{CN} > 0$. When $c = c_h^{CN}$, $\Delta_m^{CN} > 0$. It thus always holds that $\Delta_m^{CN} > 0$. ■

Proof of Theorem 2. Letting $\Delta_{m1}^{SC} \equiv \pi_m^{SN} - \pi_m^{CN}$ and solving $\Delta_{m1}^{SC} = 0$, we obtain $c_{m1}^{SC} =$

$2k\hat{\alpha}[3\hat{\alpha}+\alpha(-5+3\theta+\theta^2)]/[(1+k\hat{\alpha})(-1+\theta^2)]$. When $c < c_{m1}^{SC}$, $\Delta_{m1}^{SC} > 0$. In addition, because $c < \min\{k\hat{\alpha}[\hat{\alpha}(-3+\theta^2)+\alpha(-3+\theta+\theta^2)]/(1-\theta^2), k\hat{\alpha}[\hat{\alpha}(-11+10\theta^2-2\theta^4)+\alpha(13-7\theta-11\theta^2+4\theta^3+2\theta^4)]/(1-\theta^2)\}$ is the upper bound of scenario "CN". Let $\tilde{c}_{m1}^{CN} \equiv k\hat{\alpha}[\hat{\alpha}(-3+\theta^2)+\alpha(-3+\theta+\theta^2)]/(1-\theta^2)$ and $\tilde{c}_{m2}^{CN} \equiv k\hat{\alpha}[\hat{\alpha}(-11+10\theta^2-2\theta^4)+\alpha(13-7\theta-11\theta^2+4\theta^3+2\theta^4)]/(1-\theta^2)$. Solve $\partial\tilde{c}_{m1}^{CN}/\partial\hat{\alpha}=0$, we have $\hat{\alpha}=\alpha(3-\theta-\theta^2)/[2(5-2\theta^2)]$. When $\hat{\alpha}<\alpha(3-\theta-\theta^2)/[2(5-2\theta^2)]$, $\tilde{c}_{m1}^{CN}$ decreases with $\hat{\alpha}$, and when $\hat{\alpha}>\alpha(3-\theta-\theta^2)/[2(5-2\theta^2)]$, $\tilde{c}_{m1}^{CN}$ increases with $\hat{\alpha}$.

Similarly, when $\hat{\alpha}<\alpha(13-7\theta-11\theta^2+4\theta^3+2\theta^4)/[2(11-10\theta^2+2\theta^4)]$, $\tilde{c}_{m2}^{CN}$ increases with $\hat{\alpha}$; and, when $\hat{\alpha}>\alpha(13-7\theta-11\theta^2+4\theta^3+2\theta^4)/[2(11-10\theta^2+2\theta^4)]$, $\tilde{c}_{m2}^{CN}$ decreases with $\hat{\alpha}$. In addition, let $\tilde{c}_{m1}^{CN}=\tilde{c}_{m2}^{CN}$, we have $\hat{\alpha}=\alpha(4-2\theta-\theta^2)/(4-\theta^2)$. Note that $\alpha(4-2\theta-\theta^2)/(4-\theta^2)>\alpha(3-\theta-\theta^2)/[2(5-2\theta^2)]$ and $\alpha(4-2\theta-\theta^2)/(4-\theta^2)>\alpha(13-7\theta-11\theta^2+4\theta^3+2\theta^4)/[2(11-10\theta^2+2\theta^4)]$. Therefore, c_h^{CN} first increases and then decreases with $\hat{\alpha}$. When $\hat{\alpha}=\alpha(4-2\theta-\theta^2)/(4-\theta^2)$, c_h^{CN} reaches its maximum. However, $c_{m1}^{SC}>c_h^{CN}$ when $\hat{\alpha}=\alpha(4-2\theta-\theta^2)/(4-\theta^2)$. Therefore, $\Delta_{m1}^{SC}>0$ and the CPC payment scheme is worse than the CPS payment scheme. ■

Proof of Theorem 3. Letting $\Delta_{m2}^{SC} \equiv \pi_m^{SS}-\pi_m^{CC}$ and solving $\Delta_{m2}^{SC}=0$, we find that $\Delta_{m2}^{SC}>0$ when $c<c_{m2}^{SC}\equiv 2k\hat{\alpha}(\hat{\alpha}+\alpha)/(1+k\hat{\alpha})$. ■

Proof of Proposition 3. We compute

$$\frac{\partial p_m^{NS}}{\partial\hat{\alpha}}=\frac{5}{8}>0, \frac{\partial w^{NS}}{\partial\hat{\alpha}}=\frac{5\theta}{8}>0,\ \frac{\partial p_e^{NS}}{\partial\hat{\alpha}}=\frac{3}{2}>0, \text{ and } \frac{\partial p_r^{NS}}{\partial\hat{\alpha}}=\frac{5\theta}{8}>0.$$

Next, we calculate

$$\frac{\partial p_m^{NS}}{\partial c}=\frac{1}{8},\ \frac{\partial w^{NS}}{\partial c}=\frac{\theta}{8}>0,\ \frac{\partial p_e^{NS}}{\partial c}=\frac{1}{2}>0, \text{ and} \frac{\partial p_r^{NS}}{\partial c}=\frac{\theta}{8}>0.$$

■

Proof of Proposition 4. We calculate $\partial\pi_m^{NS}/\partial\hat{\alpha}=(-3c+13\alpha-23\hat{\alpha})/16$ and find a threshold $c_1^{NS}=(13\alpha-23\hat{\alpha})/3$. If $c<c_1^{NS}$, $\partial\pi_m^{NS}/\partial\hat{\alpha}>0$; otherwise, $\partial\pi_m^{NS}/\partial\hat{\alpha}<0$. We also compute $\partial\pi_r^{NS}/\partial\hat{\alpha}=0$ and $\partial\pi_e^{NS}/\partial\hat{\alpha}=-[3(c-3\hat{\alpha}+\alpha)]/8$, and find that $\partial\pi_e^{NS}/\partial\hat{\alpha}>0$ when $c\in\left(c_l^{NS},c_h^{NS}\right)$.

Next, $\partial\pi_m^{NS}/\partial c=(c-3\hat{\alpha}+\alpha)/16$, which is negative when $c\in\left(c_l^{NS},c_h^{NS}\right)$. Then we have $\partial\pi_r^{NS}/\partial c=0$. In addition, $\partial\pi_e^{NS}/\partial c=(c-3\hat{\alpha}+\alpha)/8$ is negative when $c\in\left(c_l^{NS},c_h^{NS}\right)$. ■

Proof of Proposition 5. We compute $\partial p_m^{NC}/\partial\hat{\alpha}=(5k\hat{\alpha}^2-c)/(8k\hat{\alpha}^2)$, which is positive because $c<3k\hat{\alpha}^2-k\hat{\alpha}\alpha<5k\hat{\alpha}^2$. We also have

$$\frac{\partial w^{NC}}{\partial\hat{\alpha}}=\frac{\theta}{8}(5-\frac{c}{k\hat{\alpha}^2})>0 \text{ and } \frac{\partial p_r^{NC}}{\partial\hat{\alpha}}=\frac{\theta}{8}(5-\frac{c}{k\hat{\alpha}^2})>0.$$

In addition, $\partial p_e^{NC}/\partial\hat{\alpha}=k(3\hat{\alpha}-\alpha/2)$. Therefore, $\partial p_e^{NC}/\partial\hat{\alpha}>0$ if $\hat{\alpha}>\alpha/6$; otherwise, $\partial p_e^{NC}/\partial\hat{\alpha}<0$ if $\hat{\alpha}<\alpha/6$.

Next, we calculate

$$\frac{\partial p_m^{NC}}{\partial c}=\frac{1}{8k\hat{\alpha}}>0,\ \frac{\partial w^{NC}}{\partial c}=\frac{\theta}{8k\hat{\alpha}}>0,\ \frac{\partial p_e^{NC}}{\partial c}=\frac{1}{2}>0, \text{ and} \frac{\partial p_r^{NC}}{\partial c}=\frac{\theta}{8k\hat{\alpha}}>0.$$

■

Proof of Proposition 6. We calculate

$$\frac{\partial \pi_m^{NC}}{\partial \hat{\alpha}} = -\frac{c^2 + k^2\hat{\alpha}^3(23\hat{\alpha} - 13\alpha) + kc\alpha\hat{\alpha}}{16k^2\hat{\alpha}^3},$$

and then find thresholds $\hat{\alpha}^{NC} = (13 + 8\sqrt{3}\alpha)/46$, $c_1^{NC} = -k\alpha\hat{\alpha}/2 + k\hat{\alpha}/2\sqrt{\alpha^2 + 52\alpha\hat{\alpha} - 92\hat{\alpha}^2}$, and $c_1^{NC} = -k\alpha\hat{\alpha}/2 - k\hat{\alpha}/2\sqrt{\alpha^2 + 52\alpha\hat{\alpha} - 92\hat{\alpha}^2} < 0$. If $\hat{\alpha} < \hat{\alpha}^{NC}$ and $c < c_1^{NC}$, $\partial \pi_m^{NC}/\partial \hat{\alpha} > 0$; otherwise, $\partial \pi_m^{NC}/\partial \hat{\alpha} < 0$. In addition, $\partial \pi_r^{NC}/\partial \hat{\alpha} = 0$.

We also find

$$\frac{\partial \pi_e^{NC}}{\partial \hat{\alpha}} = -\frac{(c + 3k\hat{\alpha}^2)[c + k\hat{\alpha}(-3\hat{\alpha} + \alpha)]}{8k^2\hat{\alpha}^3},$$

$\partial \pi_e^{NC}/\partial \hat{\alpha} > 0$ always hold when $c \in \left(c_l^{NC}, c_h^{NC}\right)$.

Next, we find

$$\frac{\partial \pi_m^{NC}}{\partial c} = \frac{c + k\hat{\alpha}(-3\hat{\alpha} + \alpha)}{16k^2\hat{\alpha}^2},$$

which is negative when $c \in \left(c_l^{NC}, c_h^{NC}\right)$. One can also find $\partial \pi_r^{NC}/\partial c = 0$. In addition,

$$\frac{\partial \pi_e^{NC}}{\partial c} = \frac{c + k\hat{\alpha}(-3\hat{\alpha} + \alpha)}{8k^2\hat{\alpha}^2},$$

which is negative when $c \in \left(c_l^{NC}, c_h^{NC}\right)$. ■

Proof of Theorem 4. Letting $\Delta_m^{NS} \equiv \pi_m^{NN} - \pi_m^{NS}$, we solve $\Delta_m^{NS} = 0$ and obtain two thresholds as $3\hat{\alpha} - \alpha - 2\sqrt{2}\sqrt{(2\hat{\alpha} - \alpha)^2}$ and $3\hat{\alpha} - \alpha + 2\sqrt{2}\sqrt{(2\hat{\alpha} - \alpha)^2} > c_h^{NS}$. Because Δ_m^{NS} is a quadratic function of c and $\partial^2 \Delta_m^{NS}/\partial c^2 = -1/16$, Δ_m^{NS} first increases and then decreases in c. Therefore, if $c \in \left(c_l^{NS}, c_\Delta^{NS}\right)$ where $c_\Delta^{NS} \equiv 3\hat{\alpha} - \alpha - 2\sqrt{2}\sqrt{(2\hat{\alpha} - \alpha)^2}$, then $\Delta_m^{NS} < 0$; otherwise, if $c \in \left(c_\Delta^{NS}, c_h^{NS}\right)$, then $\Delta_m^{NS} > 0$.

Letting $\Delta_m^{NC} \equiv \pi_m^{NN} - \pi_m^{NC}$, we solve $\Delta_m^{NC} = 0$ and obtain two thresholds as $k\hat{\alpha}(3\hat{\alpha} - \alpha) - 2\sqrt{2}k\hat{\alpha}\sqrt{(2\hat{\alpha} - \alpha)^2}$ and $k\hat{\alpha}(3\hat{\alpha} - \alpha) + 2\sqrt{2}k\hat{\alpha}\sqrt{(2\hat{\alpha} - \alpha)^2} > c_h^{NC}$. Because Δ_m^{NC} is a quadratic function of c and $\partial^2 \Delta_m^{NC}/\partial c^2 = -1/(16k^2\hat{\alpha}^2)$, Δ_m^{NC} first increases and then decreases in c. Therefore, if $c \in \left(c_l^{NC}, c_\Delta^{NC}\right)$ where $c_\Delta^{NC} \equiv k\hat{\alpha}(3\hat{\alpha} - \alpha) - 2\sqrt{2}k\hat{\alpha}\sqrt{(2\hat{\alpha} - \alpha)^2}$, then $\Delta_m^{NC} < 0$; otherwise, if $c \in \left(c_\Delta^{NC}, c_h^{NC}\right)$, then $\Delta_m^{NC} > 0$. ■

Proof of Theorem 5. Letting $\Delta_m^{SC} \equiv \pi_m^{NS} - \pi_m^{NC}$, we solve $\Delta_m^{SC} = 0$ and obtain a threshold as $c = 2k\hat{\alpha}(3\hat{\alpha} - \alpha)/(1 + k\hat{\alpha})$. When $c < 2k\hat{\alpha}(3\hat{\alpha} - \alpha)/(1 + k\hat{\alpha})$, $\Delta_m^{SC} > 0$. In addition, because $2k\hat{\alpha}(3\hat{\alpha} - \alpha)/(1 + k\hat{\alpha}) > k\hat{\alpha}(3\hat{\alpha} - \alpha)$, or, $2k\hat{\alpha}(3\hat{\alpha} - \alpha)/(1 + k\hat{\alpha}) > c_h^{NC}$, we have $\Delta_m^{SC} > 0$. ■

Proof of Proposition 7 . Scenaio NN is the win-win situation when $\pi_r^{NN} > \pi_r^{SN}$ and $\pi_m^{NN} > \pi_m^{NS}$, Scenaio NS is the win-win situation when $\pi_r^{NS} > \pi_r^{SS}$ and $\pi_m^{NS} > \pi_m^{NN}$, Scenaio SN is the win-win situation when $\pi_r^{SN} > \pi_r^{NN}$ and $\pi_m^{SN} > \pi_m^{SS}$, and Scenaio SS is the win-win situation when $\pi_r^{SS} > \pi_r^{NS}$

and $\pi_m^{SS} > \pi_m^{SN}$. Specifically, by solving $\pi_r^{NN} - \pi_r^{SN} = 0$, we have two thresholds

$$\tilde{c}_{r1}^{NN} = \frac{-3\alpha\theta^4 - 5\alpha\theta^3 + 14\alpha\theta^2 + 5\alpha\theta - 11\alpha + \left(4\theta^4 - 17\theta^2 + 13\right)\hat{\alpha} - 2\left(1-\theta^2\right)\sqrt{\tilde{H}_1}}{\left(\theta^2-1\right)^2},$$
$$\tilde{c}_{r2}^{NN} = \frac{-3\alpha\theta^4 - 5\alpha\theta^3 + 14\alpha\theta^2 + 5\alpha\theta - 11\alpha + \left(4\theta^4 - 17\theta^2 + 13\right)\hat{\alpha} + 2\left(1-\theta^2\right)\sqrt{\tilde{H}_1}}{\left(\theta^2-1\right)^2},$$

where $\tilde{H}_1 = \alpha^2\left(3\theta^4 + 14\theta^3 - 19\theta^2 - 44\theta + 50\right) + \left(8\theta^4 - 52\theta^2 + 80\right)\hat{\alpha}^2 - 4\alpha\left(3\theta^4 + 5\theta^3 - 20\theta^2 - 14\theta + 32\right)\hat{\alpha}$.

Moreover, due to $\partial^2(\pi_r^{NN} - \pi_r^{SN})/\partial c^2 = -\left(1-\theta^2\right)/\left[32\left(2-\theta^2\right)\right] < 0$, then $\pi_r^{NN} - \pi_r^{SN} > 0$ when $\tilde{c}_{r1}^{NN} < c < \tilde{c}_{r2}^{NN}$. Similarly, by solving $\pi_m^{NN} - \pi_m^{NS} = 0$, we have two thresholds $\tilde{c}_{m1}^{NN} = -\alpha - 2\sqrt{2}\sqrt{(\alpha-2\hat{\alpha})^2} + 3\hat{\alpha}$, $\tilde{c}_{m2}^{NN} = -\alpha + 2\sqrt{2}\sqrt{(\alpha-2\hat{\alpha})^2} + 3\hat{\alpha}$. Due to $\partial^2(\pi_m^{NN} - \pi_m^{NS})/\partial c^2 = -1/16 < 0$, then $\pi_m^{NN} - \pi_m^{NS} > 0$ when $\tilde{c}_{m1}^{NN} < c < \tilde{c}_{m2}^{NN}$. Taken together, one can see that Scenaio NN is the win-win situation when $c \in (\tilde{c}_{r1}^{NN}, \tilde{c}_{r2}^{NN}) \cap (\tilde{c}_{m1}^{NN}, \tilde{c}_{m2}^{NN})$.

In addition, by solving $\pi_r^{NS} - \pi_r^{SS} = 0$, we have two thresholds

$$\tilde{c}_{r1}^{NS} = \frac{-\alpha\left(\theta^2 + 26\theta + 37\right) - 2\sqrt{2}(\theta+3)\sqrt{\tilde{H}_2} + \left(-\theta^2 + 22\theta + 43\right)\hat{\alpha}}{(1-\theta)(\theta+3)},$$
$$\tilde{c}_{r2}^{NS} = \frac{-\alpha\left(\theta^2 + 26\theta + 37\right) + 2\sqrt{2}(\theta+3)\sqrt{\tilde{H}_2} + \left(-\theta^2 + 22\theta + 43\right)\hat{\alpha}}{(1-\theta)(\theta+3)},$$

where $\tilde{H}_2 = \alpha^2\left(\theta^2 + 6\theta + 25\right) + 8(5-\theta)\hat{\alpha}^2 - 64\alpha\hat{\alpha}$.

Moreover, due to $\partial^2(\pi_r^{NS} - \pi_r^{SS})/\partial c^2 = (\theta-1)/[64(\theta+1)] < 0$, then $\pi_r^{NS} - \pi_r^{SS} > 0$ when $\tilde{c}_{r1}^{NS} < c < \tilde{c}_{r2}^{NS}$. Similarly, from the previous analysis, $\pi_m^{NS} - \pi_m^{NN} > 0$ when $c < \tilde{c}_{m1}^{NN}$ or $c > \tilde{c}_{m2}^{NN}$. Taken together, one can see that Scenaio NS is the win-win situation when $c \in (\tilde{c}_{r1}^{NS}, \tilde{c}_{r2}^{NS}) \cap \{(0, \tilde{c}_{m1}^{NN}) \cup (\tilde{c}_{m2}^{NN}, 1)\}$. ■

Proof of Proposition 8 . Scenaio NN is the win-win situation when $\pi_r^{NN} > \pi_r^{CN}$ and $\pi_m^{NN} > \pi_m^{NC}$, Scenaio NC is the win-win situation when $\pi_r^{NC} > \pi_r^{CC}$ and $\pi_m^{NC} > \pi_m^{NN}$, Scenaio CN is the win-win situation when $\pi_r^{CN} > \pi_r^{NN}$ and $\pi_m^{CN} > \pi_m^{CC}$, and Scenaio CC is the win-win situation when $\pi_r^{CC} > \pi_r^{NC}$ and $\pi_m^{CC} > \pi_m^{CN}$. Specifically, by solving $\pi_r^{NN} - \pi_r^{CN} = 0$, we have two thresholds

$$\tilde{c}_{r3}^{NN} = \frac{k\hat{\alpha}\left(-\alpha(11-\theta(3\theta+5)) - 2\sqrt{\tilde{H}_3} + \left(4\theta^2-13\right)\hat{\alpha}\right)}{1-\theta^2},$$
$$\tilde{c}_{r4}^{NN} = \frac{k\hat{\alpha}\left(-\alpha(11-\theta(3\theta+5)) + 2\sqrt{\tilde{H}_3} + \left(4\theta^2-13\right)\hat{\alpha}\right)}{1-\theta^2},$$

where $\tilde{H}_3 = \alpha^2(\theta(\theta(\theta(3\theta+14)-19)-44)+50) + \left(8\theta^4 - 52\theta^2 + 80\right)\hat{\alpha}^2 - 4\alpha(\theta(\theta(\theta(3\theta+5)-20)-14)+32)\hat{\alpha}$.

Moreover, due to $\partial^2(\pi_r^{NN} - \pi_r^{CN})/\partial c^2 = -\left[\left(\theta^2-1\right)^2\right]/\left[32\left(\theta^4 - 3\theta^2 + 2\right)k^2\hat{\alpha}^2\right] < 0$, then $\pi_r^{NN} - \pi_r^{CN} > 0$ when $\tilde{c}_{r3}^{NN} < c < \tilde{c}_{r4}^{NN}$. Similarly, by solving $\pi_m^{NN} - \pi_m^{NC} = 0$, we have two thresholds $\tilde{c}_{m3}^{NN} = k\hat{\alpha}(3\hat{\alpha} - \alpha) - 2\sqrt{2}k\hat{\alpha}\sqrt{(\alpha-2\hat{\alpha})^2}$, $\tilde{c}_{m4}^{NN} = k\hat{\alpha}(3\hat{\alpha} - \alpha) + 2\sqrt{2}k\hat{\alpha}\sqrt{(\alpha-2\hat{\alpha})^2}$. Due to $\frac{\partial^2(\pi_m^{NN} - \pi_m^{NC})}{\partial c^2} = -\frac{1}{16k^2\hat{\alpha}^2} < 0$, then $\pi_m^{NN} - \pi_m^{NC} > 0$ when $\tilde{c}_{m3}^{NN} < c < \tilde{c}_{m4}^{NN}$. Taken together, one can see that Scenaio NN is the win-win situation when $c \in (\tilde{c}_{r3}^{NN}, \tilde{c}_{r4}^{NN}) \cap (\tilde{c}_{m3}^{NN}, \tilde{c}_{m4}^{NN})$.

In addition, by solving $\pi_r^{NC} - \pi_r^{CC} = 0$, we have two thresholds

$$\begin{aligned}\tilde{c}_{r1}^{NC} &= \frac{-2\sqrt{2}(\theta+3)k\hat{\alpha}\sqrt{\tilde{H}_4} - k\hat{\alpha}\left(\alpha\left(\theta^2+26\theta+37\right)+\left(\theta^2-22\theta-43\right)\hat{\alpha}\right)}{(1-\theta)(\theta+3)},\\ \tilde{c}_{r2}^{NC} &= \frac{2\sqrt{2}(\theta+3)k\hat{\alpha}\sqrt{\tilde{H}_4} - k\hat{\alpha}\left(\alpha\left(\theta^2+26\theta+37\right)+\left(\theta^2-22\theta-43\right)\hat{\alpha}\right)}{(1-\theta)(\theta+3)},\end{aligned}$$

where $\tilde{H}_4 = \alpha^2\left(\theta^2+6\theta+25\right)+8(5-\theta)\hat{\alpha}^2-64\alpha\hat{\alpha}$.

Moreover, due to $\partial^2(\pi_r^{NC}-\pi_r^{CC})/\partial c^2 = (\theta-1)\,/\left[64(\theta+1)k^2\hat{\alpha}^2\right] < 0$, then $\pi_r^{NC}-\pi_r^{CC} > 0$ when $\tilde{c}_{r1}^{NC} < c < \tilde{c}_{r2}^{NC}$. Similarly, from the previous analysis, $\pi_m^{NC}-\pi_m^{NN} > 0$ when $c < \tilde{c}_{m3}^{NN}$ or $c > \tilde{c}_{m4}^{NN}$. Taken together, one can see that Scenaio NC is the win-win situation when $c \in (\tilde{c}_{r1}^{NC}, \tilde{c}_{r2}^{NC}) \cap \{(0, \tilde{c}_{m3}^{NN}) \cup (\tilde{c}_{m4}^{NN}, 1)\}$. ■

Proof of Corollary 1. By first order condition

$$\frac{\partial\bar{\pi}_m^{CC}}{\partial d_o} = \frac{c(\theta+3)+2(\theta+3)k\hat{\alpha}d_o - k\hat{\alpha}(\alpha(\theta+19)+(\theta-13)\hat{\alpha})}{16(\theta+1)k\hat{\alpha}} = 0,$$

we get a threshold

$$d_{om} = \frac{1}{2}\left(\frac{\alpha(\theta+19)+(\theta-13)\hat{\alpha}}{\theta+3} - \frac{c}{k\hat{\alpha}}\right).$$

It is easily obtained that $\partial\bar{\pi}_m^{CC}/\partial d_o < 0$ when $d_o < d_{om}$; whereas $\partial\bar{\pi}_m^{CC}/\partial d_o > 0$ when $d_o > d_{om}$.

By first order condition

$$\frac{\partial\bar{\pi}_r^{CC}}{\partial d_o} = \frac{-c\left(\theta^2+2\theta-3\right)-2\left(\theta^2+2\theta-3\right)k\hat{\alpha}d_o + k\hat{\alpha}\left(\alpha\left(\theta^2-6\theta-59\right)+\left(\theta^2+10\theta+53\right)\hat{\alpha}\right)}{32(\theta+1)(\theta+3)k\hat{\alpha}} = 0,$$

we get a threshold

$$d_{or} = \frac{1}{2}\left(\frac{\alpha\left(\theta^2-6\theta-59\right)+\left(\theta^2+10\theta+53\right)\hat{\alpha}}{\theta^2+2\theta-3} - \frac{c}{k\hat{\alpha}}\right).$$

Furthermore, it is easily obtained that $\partial\bar{\pi}_r^{CC}/\partial d_o > 0$ when $d_o < d_{or}$; whereas $\partial\bar{\pi}_r^{CC}/\partial d_o < 0$ when $d_o > d_{or}$.

By first order condition

$$\frac{\partial\bar{\pi}_e^{CC}}{\partial d_o} = \frac{c(\theta+3)+2(\theta+3)k\hat{\alpha}d_o - k\hat{\alpha}(\alpha(\theta-5)+(\theta+11)\hat{\alpha})}{8(\theta+1)k\hat{\alpha}} = 0,$$

we get a threshold

$$d_{oe} = \frac{1}{2}\left(\frac{\alpha(\theta-5)+(\theta+11)\hat{\alpha}}{\theta+3} - \frac{c}{k\hat{\alpha}}\right).$$

It is easily obtained that $\partial\bar{\pi}_e^{CC}/\partial d_o < 0$ when $d_o < d_{oe}$; whereas $\partial\bar{\pi}_e^{CC}/\partial d_o > 0$ when $d_o > d_{oe}$. ■

Proof of Corollary 2. By first order condition

$$\frac{\partial \breve{\pi}_m^{SS}}{\partial t}=\frac{\alpha\theta^2+18\alpha\theta-19\alpha-c\left(\theta^2+2\theta-3\right)+\left(\theta^2-14\theta+13\right)\hat{\alpha}+\theta^2 t-33t}{32(\theta-1)(\theta+3)}=0,$$

we get a threshold

$$t_m=\frac{(\theta-1)(-\alpha(\theta+19)+c(\theta+3)+(\theta-13)(-\hat{\alpha}))}{\theta^2-33}.$$

Furthermore, it is easily obtained that $\partial\breve{\pi}_m^{SS}/\partial t<0$ when $t<t_m$; whereas $\partial\breve{\pi}_m^{SS}/\partial t>0$ when $t>t_m$.

By first order condition

$$\begin{aligned}\frac{\partial \breve{\pi}_r^{SS}}{\partial t} &= \frac{1}{64(1-\theta)(\theta+3)^2}[\alpha\theta^3+\alpha\theta^2+27\alpha\theta-29\alpha-c\left(\theta^3+9\theta^2+11\theta-21\right)\\ &\quad +\left(\theta^3+17\theta^2-5\theta-13\right)\hat{\alpha}+\theta^3 t+15\theta^2 t+63\theta t+49t]\\ &= 0,\end{aligned}$$

we get a threshold

$$t_r=\frac{(\theta-1)\left(-\alpha\left(\theta^2+2\theta+29\right)+c\left(\theta^2+10\theta+21\right)-\left(\theta^2+18\theta+13\right)\hat{\alpha}\right)}{(\theta+1)(\theta+7)^2}.$$

It is easily obtained that $\partial\breve{\pi}_r^{SS}/\partial t<0$ when $t<t_r$; whereas $\partial\breve{\pi}_r^{SS}/\partial t>0$ when $t>t_r$.

By first order condition

$$\frac{\partial \breve{\pi}_e^{SS}}{\partial t}=\frac{-5\alpha-c(\theta+3)+\theta(\alpha+\hat{\alpha}+t)+11\hat{\alpha}+t}{16(\theta+3)}=0,$$

we obtain a threshold

$$t_e=\frac{\alpha(5-\theta)+c(\theta+3)-(\theta+11)\hat{\alpha}}{\theta+1}.$$

Thus, it is easily obtained that $\partial\breve{\pi}_e^{SS}/\partial t<0$ when $t<t_e$; whereas $\partial\breve{\pi}_e^{SS}/\partial t>0$ when $t>t_e$. ■

Proof of Corollary 3. Let $\Psi_1=\pi_m^{SC}-\pi_m^{SS}$, we can easily find that Ψ_1 is a quadratic function with respect to c. Due to

$$\frac{\partial^2\Psi_1}{\partial c^2}=\frac{2-2k\hat{\alpha}\theta-\theta^2-k^2\hat{\alpha}^2(2-2\theta-\theta^2)}{32k^2\hat{\alpha}^2\left(1-\theta^2\right)}>0$$

and $\triangle>0$, there exist two thresholds ϕ_1 and ϕ_2 such that $\Psi_1<0$ when $\phi_1<c<\phi_2$.

Let $\Psi_2=\pi_m^{CS}-\pi_m^{CC}$, we can easily find that Ψ_2 is a quadratic function with respect to c. When $\sqrt{3}-1<\theta\leq\left(\sqrt{2\hat{\alpha}^2+4\hat{\alpha}+3}-1\right)/\left(\hat{\alpha}+1\right)$ and $0<k<\left(-\theta^2-2\theta+2\right)/\left(\theta^2\hat{\alpha}-2\hat{\alpha}\right)$, or $\left(\sqrt{2\hat{\alpha}^2+4\hat{\alpha}+3}-1\right)/\left(\hat{\alpha}+1\right)<\theta<1$,

$$\frac{\partial^2\Psi_2}{\partial c^2}=\frac{(k\hat{\alpha}-1)\left(\theta^2+2\theta+\left(\theta^2-2\right)k\hat{\alpha}-2\right)}{32k^2\hat{\alpha}^2\left(\theta^2-1\right)}>0$$

and $\triangle>0$, there exist two thresholds ϕ_3 and $\phi_3^{'}(\phi_3^{'}<0)$ such that $\Psi_2>0$ when $c>\phi_3$.

Let $\Psi_3 = \pi_r^{CS} - \pi_r^{SS}$, we can easily find that Ψ_3 is a quadratic function with respect to c. Due to

$$\frac{\partial^2 \Psi_3}{\partial c^2} = \frac{(1-k\hat{\alpha})((1-2\theta)k\hat{\alpha}+1)}{64\left(1-\theta^2\right)k^2\hat{\alpha}^2} > 0$$

and $\triangle > 0$, there exist two thresholds ϕ_4 and $\phi_4^{'}(\phi_4^{'} < 0)$ such that $\Psi_3 > 0$ when $c > \phi_4$.

Let $\Psi_4 = \pi_r^{SC} - \pi_r^{CC}$, we can easily find that Ψ_4 is a quadratic function with respect to c. When $1/2 < \theta \leq (1+\hat{\alpha})/2$ and $0 < k < (2\theta - 1)/\hat{\alpha}$, or $\theta > (1+\hat{\alpha})/2$,

$$\frac{\partial^2 \Psi_4}{\partial c^2} = \frac{(1-k\hat{\alpha})(2\theta - k\hat{\alpha} - 1)}{64\left(1-\theta^2\right)k^2\hat{\alpha}^2} > 0$$

and $\triangle > 0$, there exist two thresholds ϕ_5 and ϕ_6 such that $\Psi_4 < 0$ when $\phi_5 < c < \phi_6$. ■

Proof of Proposition 9. Since

$$\pi_r^{SN} - \pi_r^{CN} = \frac{c(1-k\hat{\alpha})(c(1+k\hat{\alpha})(-1+\theta^2) + 2k\hat{\alpha}(\hat{\alpha}(13-4\theta^2) - \alpha(11-5\theta-3\theta^2)))}{64k^2\hat{\alpha}^2(2-\theta^2)},$$

we can get a threshold $2k\hat{\alpha}\left[\hat{\alpha}(13-4\theta^2) - \alpha(11-5\theta-3\theta^2)\right]/(1+k\hat{\alpha})(1-\theta^2)$.

When $c < 2k\hat{\alpha}\left[\hat{\alpha}(13-4\theta^2) - \alpha(11-5\theta-3\theta^2)\right]/(1+k\hat{\alpha})(1-\theta^2)$, $\pi_r^{SN} > \pi_r^{CN}$; otherwise, $\pi_r^{SN} < \pi_r^{CN}$. Since

$$\pi_r^{SS} - \pi_r^{CC} = \frac{c(-1+k\hat{\alpha})(2k\hat{\alpha}(\alpha(\theta^2+26\theta+37) + \hat{\alpha}(\theta^2-22\theta-43)) - c(1+k\hat{\alpha})(-3+2\theta+\theta^2))}{128k^2\hat{\alpha}^2(1+\theta)(3+\theta)},$$

we can get a threshold $2k\hat{\alpha}[\hat{\alpha}(43-\theta^2+22\theta) - \alpha(\theta^2+26\theta+37)]/(1+k\hat{\alpha})(1-\theta)(\theta+3)$. When $c < 2k\hat{\alpha}[\hat{\alpha}(43-\theta^2+22\theta) - \alpha(\theta^2+26\theta+37)]/(1+k\hat{\alpha})(1-\theta)(\theta+3)$, $\pi_r^{SS} > \pi_r^{CC}$; otherwise, $\pi_r^{SS} < \pi_r^{CC}$.
■

Proof of Proposition 10. We find that

$$\pi_e^{NS} - \pi_e^{NC} = \frac{c(1-k\hat{\alpha})(2k\hat{\alpha}(3\hat{\alpha}-\alpha) - c(1+k\hat{\alpha}))}{16k^2\hat{\alpha}^2} > 0,$$

$$\pi_e^{SN} - \pi_e^{CN} = \frac{c(1-k\hat{\alpha})(c(1+k\hat{\alpha})(-1+\theta^2) + 2k\hat{\alpha}(\hat{\alpha}(5-2\theta^2) + \alpha(-3+\theta+\theta^2)))}{16k^2\hat{\alpha}^2(2-\theta^2)} > 0,$$

and

$$\pi_e^{SS} - \pi_e^{CC} = \frac{c(1-k\hat{\alpha})(2k\hat{\alpha}(\hat{\alpha}(11+\theta) + \alpha(-5+\theta)) - c(1+k\hat{\alpha})(3+\theta))}{32k^2\hat{\alpha}^2(1+\theta)} > 0$$

always hold throughout the domain. ■